Mixed convection heat transfer for nanofluids in a lid-driven shallow rectangular cavity uniformly heated from below

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Abstract

This paper reports an analytical and numerical study of natural convection in a shallow lid-driven rectangular cavity filled with nanofluids. Neumann boundary conditions for temperature are applied to the horizontal walls of the enclosure, while the two vertical ones are assumed insulated. The governing parameters for the problem are the thermal Richardson number, Ri, the aspect ratio of the cavity, A, the Reynolds number, Re, and the solid volume fraction of Cu-nanoparticles Φ (Pr = 7). Its has been performed numerically by solving the full governing equations via the finite volume method and the SIMPLER algorithm. In the limit of a shallow cavity for convection in an infinite layer, analytical solutions for the stream function and temperature are obtained using a parallel flow approximation in the core region of the cavity and an integral form of the energy equation.

Keywords: *nanofluid; mixed convection; lid-driven cavity;parallel flow; finite volume method.*

1. Introduction

Heat transfer with conventional fluids, such as water and oil, whose thermal conductivity is inherently poor, is limited, which is a crucial problem to challenge. In such a context, Choi (1995), of Argonne National Laboratory, developed the novel concept of nanofluids as a route to improve the performances of heat transfer fluids currently available. This new class of advanced heat transfer fluids is engineered by dispersing solid nanoparticles (metallic, non-metallic or polymeric), smaller than 100 nm in diameter, in base fluids (aqueous or organic host liquids), which confers a large thermal conductivity on these ones and makes them potentially useful in engineering equipments involving heat transfer. Numerous studies, on convection heat transfer, have been conducted, and most of them have dealt with forced convection, indicating that nanoparticle suspensions have unquestionably a great potential for heat transfer enhancement, as reported in a recent paper by Corcione (2010). At the same time, mixed convection has not received either less attention in view of the number of the related works recently done. Among them, flow and heat transfer problem in lid-driven cavities, which finds applications in industrial processes such as food processing, float glass production (Pilkington, 1969), thermal-hydraulics of nuclear reactors (Ideriah, 1980), dynamics of lakes (Imberger, Hamblin, 1982), crystal growth, flow and heat transfer in solar ponds (Cha and Jaluria, 1984), lubrication technologies (Tiwari and Das, 2007) and so on. The interaction of the shear driven flow due to the lid motion and natural convective flow due to the buoyancy effect is quite complex, which necessitates a comprehensive analysis to understand the physics of the resulting flow and heat transfer process. In this respect, different configurations and combinations of thermal and dynamical boundary conditions have been considered and analyzed by some investigators. The present paper focuses attention on such a problem within a two-dimensional shallow rectangular enclosure, filled with *Cu*–water nanofluids, whose short vertical sides are submitted to uniform heat fluxes while the long horizontal ones are maintained adiabatic with the top moving in the opposite direction to the heat flux. A numerical solution of the full governing equations is obtained via a finite volume method. An analytical one, based on the parallel flow approximation, is also proposed. The results are presented, for various values of the dimensionless parameters, controlling the problem, which are the Reynolds, *Re*, and Richardson, *Ri*, numbers, and the solid volume fraction of nanoparticles, *Φ*.

2. Mathematical formulation

The studied configuration is sketched in Fig. 1. It is a shallow rectangular enclosure of height H' and length L', filled with *Cu*–water nanofluids. The long horizontal walls are submitted to a uniform density of heat flux, q', while the vertical walls are adiabatic.

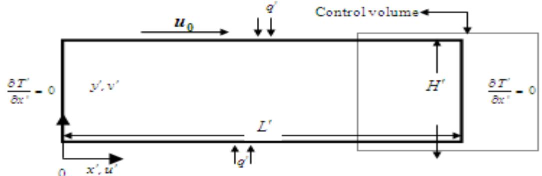


Fig. 1: Geometry and coordinates system

The dimensionless governing equations and the corresponding boundary conditions are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\bar{\rho}}\frac{\partial P}{\partial x}+\frac{\bar{\nu}}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\bar{\rho}}\frac{\partial P}{\partial y}+\frac{\bar{\nu}}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{\bar{\beta}}{\bar{\rho}}RiT \quad (3)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\bar{\alpha}}{Pe}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

$$u=v=\frac{\partial T}{\partial x}=0 \text{ for } x=0 \text{ and } A \quad (5)$$

$$u=v=\frac{\partial T}{\partial y}+\frac{1}{\bar{k}}=0 \text{ for } y=0 \quad (6)$$

$$u-1=v=\frac{\partial T}{\partial y}+\frac{1}{\bar{k}}=0 \text{ for } y=1 \quad (7)$$

The above equations let appears some dimensionless parameters that govern the problem, namely, the solid volume fraction Φ, the aspect ratio of the enclosure, A, the Peclet, Pe, Reynolds, Re, and Richardson, Ri, numbers. For the last four, the expressions are

$$A=\frac{L'}{H'} \quad Pe=\frac{U_0'H'}{\alpha_f} \quad Re=\frac{U_0'H'}{\nu_f} \text{ and } Ri=\frac{g\beta_f q'H'^2}{k_f U_0'^2}$$

The local heat transfer, through the nanofluid-filled cavity, can be expressed in terms of the local Nusselt number defined as:

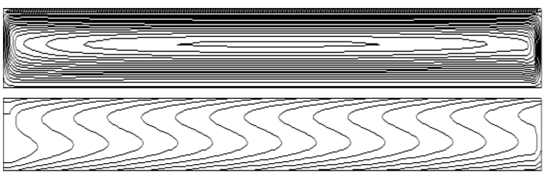
$$Nu(x)=\frac{hL'}{k_f}=\frac{q'}{\Delta T'}\frac{H'}{k_f}=\frac{\Delta T^*}{\Delta T'}=\frac{1}{\Delta T} \quad (8)$$

3. Numerics

The dimensionless governing equations have been solved by using a finite volume method and SIMPLER algorithm in a staggered uniform grid system (Patankar, 1980). The convergence has been considered as reached when $\sum_{i,j}\left|f_{i,j}^{k+1}-f_{i,j}^{k}\right|<10^{-5}\sum_{i,j}\left|f_{i,j}^{k+1}\right|$, where $f_{i,j}^{k}$ stands for the value of *u*, *v*, *p* or *T* at the k^{th} iteration level and grid location (*i*, *j*) in the plane (*x*, *y*).

4. Approximate parallel flow analytical solution

As can be seen from Figs.2, displaying streamlines (top) and isotherms (low), the flow and temperature fields exhibit a parallel aspect and a linear Stratification, respectively, in the most part of the cavity, for $A=14$ and various values of *Re*, *Ri* and *Φ*.



Figs. 2 Streamlines (top) and isotherms (low) for $A=14$, $\mathrm{Re}=10$ and $Ri=10^3$ various values of Φ

Accordingly, the following simplifications $u(x,y)=u(y)$, $v(x,y)=0$, $\psi(x,y)=\psi(y)$ and $T(x,y)=C(x-A/2)+\theta(y)$

Where C is unknown constant temperature gradient in the x-direction.

The ordinary non-dimensional governing equations:

$$\frac{d^3u}{dy^3}=\bar{\alpha}\Omega\,\mathrm{Re}\,Ri\frac{\partial T}{\partial x}=\bar{\alpha}\Omega\,\mathrm{Re}\,RiC \quad (9)$$

$$\frac{\bar{\alpha}}{Pe}\frac{d^2\theta}{dy^2}=Cu \quad (10)$$

$$u-1=\frac{d\theta}{dy}+\frac{1}{k_{nf}}=0 \text{ for } y=0 \text{ and } 1 \quad (11)$$

$\int_0^1 u(y)dy=0$ $\int_0^1 \theta(y)dy=0$ as boundary, return flow and mean temperature conditions, respectively.

Using such an approach, the solution of Equations:

$$u(y)=\frac{\bar{\alpha}}{12}\Omega\,\mathrm{Re}\,RiC\left(2y^3-3y^2+y\right)+\left(3y^2-2y\right) \quad (12)$$

$$\theta(y)=\frac{1}{12}\Omega RaC^2\left(\frac{y^5}{10}\quad\frac{y^4}{4}+\frac{y^3}{6}\quad\frac{1}{120}\right)+\frac{PeC}{\bar{\alpha}}\left(\frac{y^4}{4}\quad\frac{y^3}{3}+\frac{1}{30}\right) \quad (13)$$

$$+\frac{1}{k_{nf}}\left(\frac{1}{2}-y\right)$$

$$\psi(y)=\frac{\bar{\alpha}}{12}\Omega\,\mathrm{Re}\,RiC\left(\frac{y^4}{2}-y^3+\frac{y^2}{2}\right)+\left(y^3-y^2\right) \quad (14)$$

Where $\Omega=\frac{\bar{\beta}}{\bar{\rho}\bar{\alpha}\bar{\nu}}$ and $Ra=Ri.Re.Pe$

On the other hand, according to Bejan (1983), the energy balance in *x*-direction is:

$$\int_0^1 -\frac{\partial T}{\partial x}dy+\frac{Pe}{\bar{\alpha}}\int_0^1 uTdy=0 \quad (15)$$

In particular, in the parallel flow region and with the application of Eq. (4), Eq. (15) becomes

$$C+\frac{Pe}{\bar{\alpha}}\int_0^1 u\theta\,dy=\frac{a}{\bar{k}}=0$$

(16)

Which, when substituted to Eqs. (12) and (13), Eq. (15) gives the following transcendental equation:

$$\frac{Pe}{12\bar{\alpha}\bar{k}}+\left(1+\frac{Pe^2}{105\bar{\alpha}^2}\quad\frac{Ra}{720\bar{\alpha}\bar{k}}\right)C+\frac{\Omega PeRa}{3360\bar{\alpha}}C^2+\frac{\Omega^2Ra^2}{362880}C^3=0 \quad (17)$$

Whose solution, via Newton-Raphson method, for given Pe, Ra and Φ, leads to C.

The Nusselt number is constant and can be expressed as:

$$Nu=\overline{Nu}=\left(-\frac{\Omega RaC^2}{720}+\frac{\mathrm{RePr}C}{12\bar{\alpha}}+\frac{1}{k_{nf}}\right)^{-1} \quad (18)$$

5. Results and discussion

With boundary conditions of Neumann kind the flow and thermal fields, and thermo-convective characteristics become parallel, stratified and independent on the enclosure aspect ratio, *A*, respectively, when this parameter tends to be large enough. In our situation, this occurs with $A = 14$ in the limit of the explored values of *Re* ($0.1 \le \mathrm{Re} \le 10$), *Ri* ($10 \le Ri \le 10^5$) and *Φ* ($0 \le \Phi \le 0.2$), and $\mathrm{Pr} = 7$ (water based mixtures). Consequently, the mixed convection flow developed within the enclosure is governed only by four dimensionless parameters, namely, *Re*, *Ri* and *Φ*.

For analysis the problem, the flow intensity (top), ψ_c , and heat transfer rate (bottom), $\overline{Nu}$, are reported, against *Ri*, in Figs. 3 and 4, for each *Re* and various *Φ*. It is easy to observe that exhibits in general two tendencies, whose expanse depends on Re and Φ.

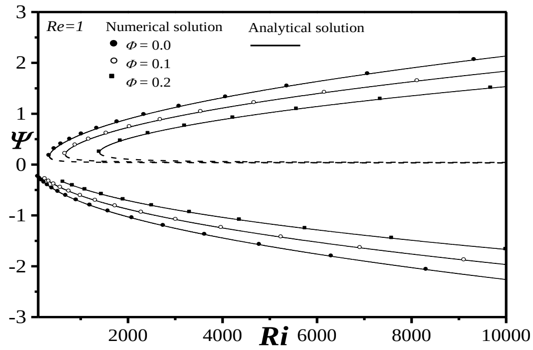


Fig.3: Variations with Ri of ψ_c for Re = 1and various values of ϕ.

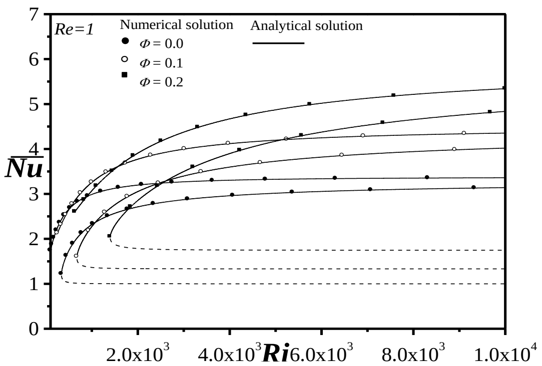


Fig.4: Variations with Ri of $\overline{Nu}$ for Re = 1and various values of ϕ**.**

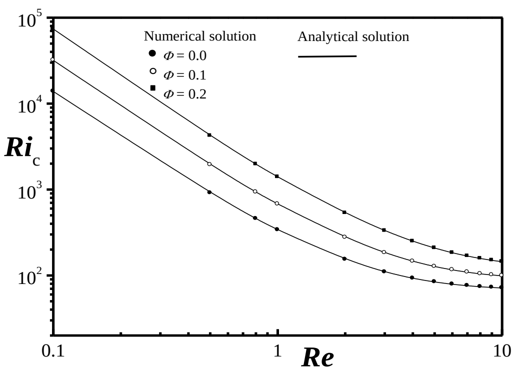


Fig. 5: Onset of the Rayleigh-Bénard convection

It is easy to observe that in fig.3 exhibits in general two distinct stable convective regimes, unicellular, and bicellular, where the quantity $|\psi_c|$ increases with Ri, for Φ and Re fixed. With regard $\overline{Nu}$, (see fig.4) to increase the number of Richardson is generally associated with an increase in the rate of heat transfer due to the shear flow.

In Fig. 5, is an illustration of the evolution of the critical Richardson number, corresponding to the occurrence of Rayleigh-Bénard convection in the presence of the forced one as a function of Re, for different values of Φ. It seems to dininute with Re to reach an asymptotic value depending on Φ.

Conclusion*:*

The study of mixed convection Rayleigh-Benard, caused by the imposition of a heat flux on the lower wall of a horizontal rectangular cavity confining a nanofluid was treated numerically and analytically. The full partial differential equations, governing the problem, have been solved numerically using a finite volume method. The computations, which have been limited to Cu-water mixtures, with $\mathrm{Pr} = 7$, have been carried out with governing parameters, Re, Ri and Φ, varying, respectively, in the ranges $0.1 \le \mathrm{Re} \le 10$, $1 \le Ri \le 10^5$ and $0 \le \Phi \le 0.2$. Analytical solution is derived on the basis of a parallel flow assumption in the core region of the enclosure. The main findings of such an investigation can be summarized as follows:

- Flow and temperature fields are strongly depend on the Richardson number, measuring the relative importance of both lid and buoyancy-driven effects.
- Increase the number of Richardson is generally associated with an increase in the rate of heat transfer due to the shear flow and an increase in the intensity of flow.
- The addition of Cu-nanoparticles in pure water leads to an improvement of heat transfer by convection and to a deterioration of the intensity of the flow.

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