Double-diffusive and Soret-induced convection in a shallow horizontal layer filled with non-Newtonian power-law fluids

K. BIHICHE[1], M. LAMSAADI[1], M. NAÏMI[1], H.ELHARFI[1], M.KADDIRI[1], A.LOUARAYCHI[1]
1. Sultan Moulay Slimane University, Faculty of Sciences and Technologies, Physics Department, Laboratory of Flows and Transfers Modelling (LAMET), B.P.523, Beni-Mellal 23000, Morocco, Emails : khadija.bihiche@gmail.com; lamsaadima@yahoo.fr; naimima@yahoo.fr; m.kadiri@fstbm.ac.ma; elharfi.hassan@yahoo.fr

Abstract

This paper reports an analytical and numerical study of the natural convection in a horizontal shallow cavity filled with non-Newtonian power-law fluids. A uniform heat flux is applied to the vertical walls while the horizontal walls are impermeable and adiabatic. The solutal buoyancy forces are assumed to be induced either by the imposition of constant fluxes of mass on the vertical walls (double-diffusive convection, a = 0) or by temperature gradients (Soret induced convection, a = 1). The case where the buoyancy forces induced by the thermal and solutal effects are opposing each other and of equal intensity (N = −1) is considered. For this situation the critical Rayleigh number for the onset of convection is predicted.

Keywords: *Double diffusive convection, finite volume method, Heat and mass transfer, Soret effect*.

1. Introduction

The thermal diffusion phenomenon, also called Soret effect, occurs in a mixture where a temperature gradient induces a transfer of solute even with an initial uniform concentration. The solute migrates toward the hot or cold side depending on the mixture composition and, sometimes, on the temperature range too [1]. Thermodiffusion in different fluids is a subject of intensive research due to its wide range of applications in many engineering and technological areas. These include geophysics, oil reservoirs, multi-component melts and storage of nuclear wastes and many other applications.

The present investigation is concerned with the case of a shallow layer of a non Newtonian power-law fluid submitted to the influence of thermal and concentration horizontal gradients. The solutal buoyancy forces are assumed to be induced either by the imposition of constant fluxes of mass on the vertical walls (double diffusive convection) or by temperature gradients (Soret effects).

An analytical model, based on the parallel flow approximation, is proposed for the case of a shallow layer (A>>1). The study is completed by a numerical solution of the full governing equations.

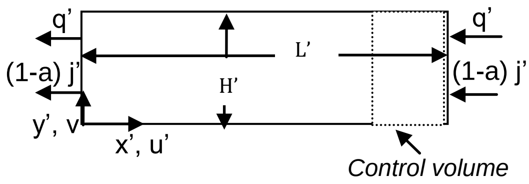

Figure 1: Sketch of the cavity and co-ordinates system.

2. Formulation mathématique

The dimensionless governing equations describing conservation of mass, momentum, and energy, written in terms of velocity components, pressure, temperature and concentration are given as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial x}+\Pr\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\Pr\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (3)$$
$$+Ra_T\ \Pr(T+NS)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2} \quad (4)$$

$$\frac{\partial S}{\partial t}+u\frac{\partial S}{\partial x}+v\frac{\partial S}{\partial y}=\frac{1}{Le}\left[\left(\frac{\partial^2 S}{\partial x^2}+\frac{\partial^2 S}{\partial y^2}\right)-a\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)\right] \quad (5)$$

The corresponding boundary conditions are

$$u=v=\psi=\frac{\partial T}{\partial y}=\frac{\partial S}{\partial y}=0 \quad \text{for } y = 0, 1 \quad (6)$$

$$\frac{\partial T}{\partial x}=1 \quad \frac{\partial S}{\partial x}=(1-a)+a\frac{\partial T}{\partial x} \quad \text{for } x = 0, A \quad (7)$$

$$u=v=\psi=0 \quad (8)$$

3. Resolution method

3.1 Numerical method

The solution of the governing equations and boundary conditions, Eqs. (1)–(8), is obtained using a control volume approach and the SIMPLER algorithm (Patankar [2]).

Typical numerical results, in terms of streamlines, isotherms and isoconcentration, are presented in Figure 2. These figures show that the flow is parallel to the longitudinal boundaries and the temperature and the concentration are linearly stratified. More discussion will focus later on this flow character, which represents the foundation of the parallel flow assumptions that allows an appropriate problem simplification.

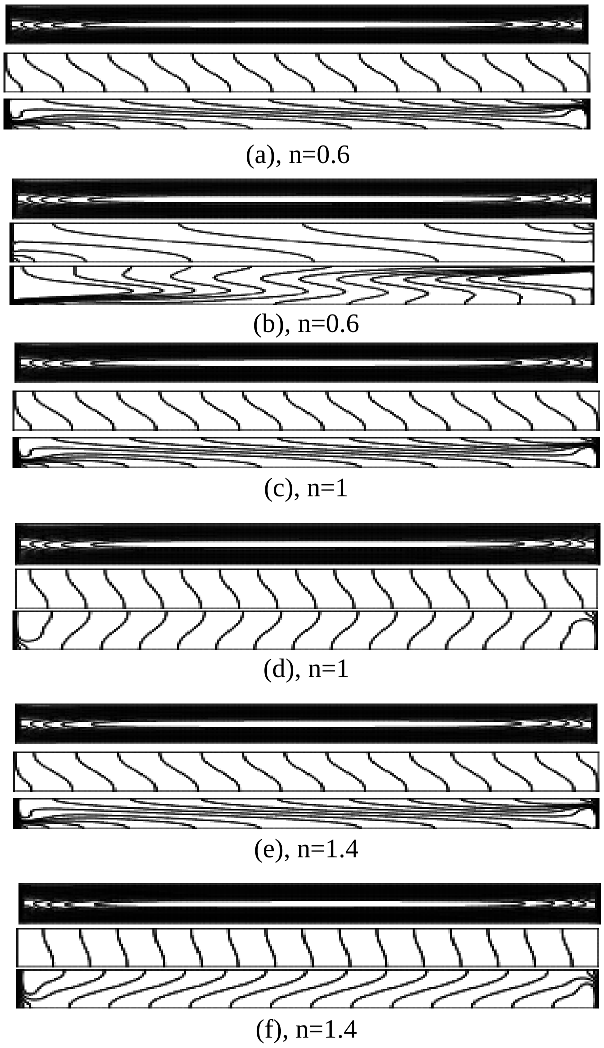

Figure 2 :Streamlines, isotherms and isoconcentrations for $Ra_T = 10^3$, N =- 1, Le = 10, and differents valeurs of n. (a),(c),(e) double diffusive convection a=0, (b),(d),(f) Soret-driven convection a=1

3.2 Parallel flow analysis

On the basis of the results presented in figure 2, the following simplifications are used:

$$u(x,y) = u(y) \qquad v(x,y) = 0 \tag{10}$$

$$T(x,y) = C_T(x - A/2) + \theta_T(y) \tag{11}$$

$$S(x,y) = C_S(x - A/2) + \theta_S(y) \tag{12}$$

Where C_T and C_S are unknown constant temperature and concentration gradients in the x-direction. The expressions of C_T and C_S can be deduced by integration of Eqs. (13) and (14), together with the boundary conditions (6) and (7), by considering the arbitrary control volume of Fig. 1 and connecting with the region of the parallel flow [3]. This yields:

$$C_S - (1-a) - aC_T = \int_0^1 u(y)\theta_S(y)dy \tag{13}$$

$$C_T + 1 = \int_0^1 u(y)\theta_T(y)dy \tag{14}$$

In such a situation the full non-dimensional governing equations reduce to:

On the basis of the results presented in figure 2, the following simplifications are used :

$$\frac{\partial^2}{\partial y^2}\left[\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right] = ERa_T \quad \text{with} \quad E = C_T + NC_S \tag{15}$$

$$C_T u = \frac{\partial^2 \theta_T}{\partial y^2} \tag{16}$$

$$(LeC_S + aC_T)u = \frac{\partial^2 \theta_S}{\partial y^2} \tag{17}$$

The integration of Eqs. (15), (16) and (17), coupled with the conditions (6), (7) and (8), leads to analytical expressions of velocity, temperature and concentration. However, such an operation is difficult to carry out owing to the particular nature of the governing equations and requires, therefore, a special numerical treatment [4].

4. Results and discussion

The fact of imposing uniform heat and mass fluxes, as boundary conditions, leads to flow characteristics independent on the aspect ratio, A, when this parameter is large enough. . The approximate solution, developed in the preceding section, on the basis of the parallel flow assumption, is thus valid asymptotically in the limit of a shallow cavity (A >>1). The critical Rayleigh numbers for the onset of convection are predicted by this theory, The rest state is a possible solution provided that the Rayleigh number is below a critical value (fig. 3).

Therefore, numerical tests are performed to determine the smallest value of A leading to results reasonably close to those of large aspect ratio approximation : the asymptotic analytical limits are largely reached for A = 24. Thus, it is clear that the flow is generally unicellular, turning clockwise, with a parallelism and horizontal heat and mass stratifications in major part of the cavity.

It can be easily demonstrated that the value of Ψ_c can be evaluated from the following expression:

$$Le^2 An^2 Dn^4 \psi_C^4 - An(1+Le^2)Dn^2\psi_C^2 - An[-Le^2 + aLe + 1 + a]RaDn^{2-n}\psi_C^{2-n} = 0$$

It is seen from the present theory that the onset of motion, occurs at a critical Rayleigh number Ra_{TC} given by

$$Ra_{TC} = \frac{Le^2 An^2 Dn^4 \psi^{*4}_C - An(1+Le^2)Dn^2\psi^{*2}_C + 1}{An[-Le^2 + aLe + 1 + a]RaDn^{2-n}\psi^{*2-n}_C}$$

Where

$$\psi^*_C = [-nAn(1+Le^2)Dn^2 + [(nAn[1+Le^2]Dn^2)^2 - 4(n^2-4)An^2Le^2Dn^4]^{1/2} / 2(2+n)An^2Le^2Dn^4]^{1/2}$$

The influence of the thermal Rayleigh number, Ra_T, on the flow intensity and the heat and mass transfer rates, is illustrated on Fig. 4, for Le = 10, N = -1 for a=0 and 1.
It is observed from figure 3 that, the flow intensity increases strongly as the value of Ra_T is made larger than the Ra_{TC}.

	Ra_{TC}	
n	a = 0	a = 1
0.6	74.729	67.256
1.0	122.889	110.600
1.4	168.976	152.079

Table 1 *:* Ra_{TC} for various value of n, a = 0 double diffusive convection and a = 1 Soret-driven convection.

The analytical solution, depicted by the solid and dashed lines, indicates the possible existence of two convective modes for a given value of Ra_T . The solution corresponding to the higher convective mode (solid line), was found numerically to be stable. On the other hand it has not been possible to obtain numerical results for the lower (dashed line) unstable branch.

(a), n=0.6

(b), n=1

(c), n=1.4

Figure 3 *:* Temperature for $Ra_T = 50$, N =- 1, Le = 10, a=1 and various n.

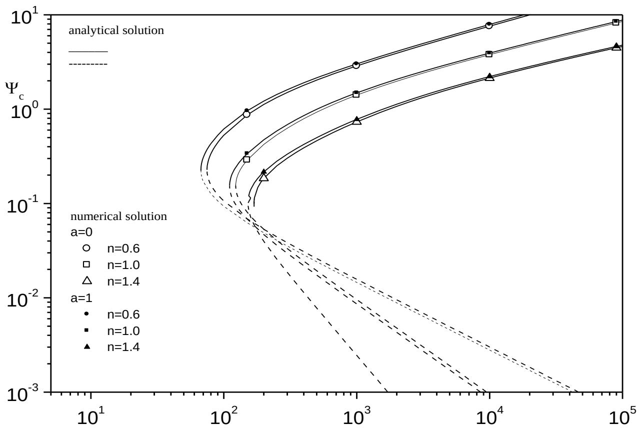

Figure 4 *:* Effect of Ra_T on stream function in the centre of the cavity for N = -1, Le = 10 , a=0 and 1 and differents valeurs of n

5. Conclusion

In this article, the problem of natural convection in a horizontal fluid layer subject to horizontal gradients of temperature and solute has been solved by both analytical and numerical methods. The influence of the thermal Rayleigh number Ra_T, and parameter a (=0, double diffusive convection; and =1 Soret induced convection) on the intensity of convection were predicted and discussed. The summary of the major results is:

i) An analytical solution, based on the parallel flow approximation, has been derived for the case of a shallow layer A>>1. Although, the resulting analytical model requires a numerical procedure to solve a transcendental equation, this is by far a much easier task than solving numerically the full set of governing equations.

ii) a steady rest state solution corresponding to a purely diffusive regime is possible. The critical Rayleigh number, Ra_{TC}, for the onset of convection is deducted from the nonlinear parallel flow approximation. In the vicinity of N $= -1$, the existence of multiple solutions, for a given set of the governing parameters is demonstrated both analytically and numerically.

Références

[1] A. Mojtabi, J.K. Platen, and M.C. Charrier-Mojtabi, *Naissance de la convection libre dans les solutions salines chauffées par le bas à coefficient de Soret variable*, Entropie, vol.218, pp.39-42,1999

[2] S. Patankar, *Num. Heat Transfer and Fluid Flow.* Hemisphere, New York (1980).

[3] T. Makayssi, M. Lamsaadi, M. Naimi, M. Hasnaoui, A. Raji, A. Bahlaoui, *Effect of solutal buoyancy forces on thermal convection in confined non-newtonian powerlaw fluids*, The International Scientific Journal for Alternative Energy and Ecology, N° 6, 62, 2008.

[4] T. Makayssi, M. Lamsaadi, M. Naimi, M. Hasnaoui, A. Raji, A. Bahlaoui, *Natural double-diffusive convection in a shallow horizontal rectangular cavity uniformly heated and salted from the side and filled with non-Newtonian power-law fluids: The cooperating case* , Energy Conversion and Management, N° 49, 2021, 2008.