

Effect of rotation on the onset of nanofluid convection in a Hele-Shaw cell

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Abstract

The nanofluid convection driven by centrifugal forces in a rotating Hele-Shaw cell is studied by using a linear stability analysis. The nanofluid model used combines the effects of Brownian motion and thermophoresis. The linear stability equations are solved numerically to find the critical values of the Rayleigh and the wave numbers. The analysis reveals that the curvature parameter and the Eckman number delay the onset of convection, while the nanoparticle concentration Rayleigh number hasten the onset of convection.

Keywords : *Stability; Nanofluid convection; Thermophoresis; Brownian motions; Rotation.*

1. Introduction

The term nanofluid refers to the suspensions of nanoscale particles in the base fluid. To study the heat transfer in nanofluid, Buongiorno [1] considered seven slip mechanisms: inertia, Brownian diffusion, thermophoresis, diffusophoresis, Magnus effect, fluid drainage and gravity settling. It was concluded that the most important slip mechanisms are Brownian diffusion and thermophoresis. Based on these two effects, he has proposed a new model of conservation equations. This model is used by several authors to study the problem related to thermal instability in nanofluids with a fixed nanoparticle volumetric fraction on the boundaries [2-4]. Recently, with a new set of boundary conditions, where the nanoparticle flux is assumed to be zero rather than prescribing the nanoparticle volumetric fraction on the boundaries, Nield et al. [5] have studied the thermal instability in a porous medium layer saturated by a nanofluid. They have shown that nanoparticle fraction value at the boundary adjusts accordingly.

Very recently Yadav et al. [6] have examined the onset of convection in nanofluid layer confined within a Hele-Shaw cell with new physical boundaries conditions where the nanoparticle fraction alters itself together with the consequence of Brownian and thermophoresis motions on the boundaries was taken.

The aim of the present investigation is to study the onset of convection, driven by centrifugal forces, in a nanofluid confined in an annular rotating Hele-Shaw cell taken into account the effects of Brownian motion and thermophoresis. We perform a linear stability analysis to investigate the effects of Coriolis force, curvature

parameters, and other parameters on the stationary nanofluid convection.

2. Formulation

Consider a nanofluid confined in an annular Hele-Shaw cell of thickness e , which is subject to a uniform angular velocity, Ω , around its vertical symmetry axis. The nanofluid is bounded horizontally by two thermally insulating walls. The inner and outer circular boundaries are maintained at constant but different temperature, T_1^* and T_2^* , respectively. We denote by, $\varepsilon=e/\Delta R \ll 1$, the aspect ratio of the cell, where $\Delta R=R_2-R_1$. The curvature parameter of the Hele-Shaw cell is defined by, $\delta= \Delta R/R_1$. In this study we neglect the gravity assuming that, $g \ll \Delta R \Omega^2$. The schematic diagram of the system considered is shown in figure 1.

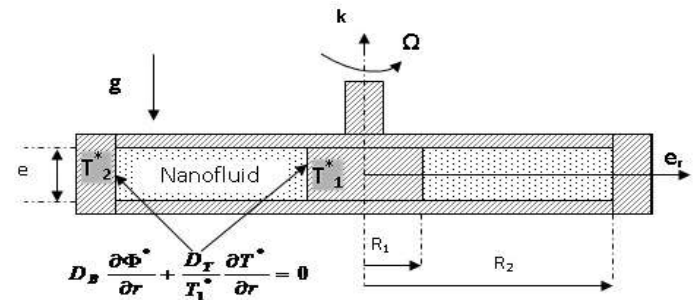


Figure 1: Physical configuration of the system considered.

In the relative frame and employing the Oberbeck-Boussinesq approximation, the governing equations to study the thermal instability in a nanofluid layer [1] taking into account the Coriolis forces are written as follow:

$$\nabla \cdot \mathbf{V}^* = 0 \quad (1)$$

$$\rho \frac{D\mathbf{V}^*}{Dt^*} = -\nabla P^* + \mu \Delta \mathbf{V}^* - \left[\rho_p \Phi^* + \rho(1 - \Phi^*) \left(1 - \beta(T^* - T_1^*) \right) \right] \Omega \wedge (\Omega \wedge \mathbf{OM}) - 2\rho \Omega \wedge \mathbf{V}^* \quad (2)$$

$$(\rho C)_f \frac{DT^*}{Dt^*} = \kappa \Delta T^* + (\rho C)_p [D_B \nabla \Phi^* \nabla T^* + \frac{D_T}{T_1^*} \nabla T^* \nabla T^*] \quad (3)$$

$$\frac{D\Phi^*}{Dt^*} = D_B \Delta \Phi^* + \frac{D_T}{T_1^*} \Delta T^* \quad (4)$$

We assume that the temperature is constant and nanoparticles flux is zero on the boundaries. Then, the boundary conditions are:

$$\mathbf{V}^* = \mathbf{0}, T^* = T_1^*, D_B \frac{\partial \Phi^*}{\partial r} + \frac{D_T}{T_1^*} \frac{\partial T^*}{\partial r} = 0 \text{ at } r^* = R_1$$

$$\mathbf{V}^* = \mathbf{0}, T^* = T_2^*, D_B \frac{\partial \Phi^*}{\partial r} + \frac{D_T}{T_1^*} \frac{\partial T^*}{\partial r} = 0 \text{ at } r^* = R_2$$

3. Linear stability analysis

The basic motionless state of the nanofluid is described by:

$$\mathbf{V}_b = \mathbf{0}, T_b(r) = \frac{\text{Log}(1 + \delta r)}{\text{Log}(1 + \delta)}$$

and

$$\Phi_b(r) = 1 - N_A \frac{\text{Log}(1 + \delta r)}{\text{Log}(1 + \delta)}$$

By superposing a perturbation to the basic solution and neglecting products of prime quantities for the case $Pr=1$ or $Pr \gg 1$. Analyzing the disturbances into the normal modes in the Hele-Shaw approximations [7] to obtain the following dimensionless system of equations:

$$D_m F(r) = - \left(\frac{1 + \delta r}{\delta} \right)^2 M_2 \left[R_a \frac{\partial^2 g}{\partial r^2} - R_n \frac{\partial^2 h}{\partial r^2} \right] - 5 \frac{1 + \delta r}{\delta} M_2 \left[R_a \frac{\partial g}{\partial r} - R_n \frac{\partial h}{\partial r} \right] - \frac{4M_2^2 + m^2 M_1^2}{M_2} [R_a g(r) - R_n h(r)] \quad (5)$$

$$D_m g(r) - R_a M_2 \frac{1 + \delta r}{\delta} \frac{\partial T_b}{\partial r} g(r) + \left[\frac{2N_A N_B}{L_e} \frac{\partial T_b}{\partial r} + \frac{N_B}{L_e} \frac{\partial \Phi_b}{\partial r} \right] \frac{\partial g}{\partial r} - im \sigma g(r) = \frac{\delta}{1 + \delta r} \frac{\partial T_b}{\partial r} F(r) - R_n M_2 \frac{1 + \delta r}{\delta} \frac{\partial T_b}{\partial r} h(r) - \frac{N_B}{L_e} \frac{\partial T_b}{\partial r} \frac{\partial h(r)}{\partial r} \quad (6)$$

$$D_m h(r) + R_n L_e M_2 \frac{1 + \delta r}{\delta} \frac{\partial T_b}{\partial r} h(r) - im \sigma L_e h(r) = L_e \frac{\partial \Phi_b}{\partial r} \frac{\delta}{1 + \delta r} F(r) + \left[R_a L_e M_2 \frac{1 + \delta r}{\delta} - N_A D_m \right] g(r) \quad (7)$$

This system is subject to the following simplified boundary conditions:

$$F = g = \frac{\partial h}{\partial r} + N_A \frac{\partial g}{\partial r} = 0 \text{ for } r = 0, 1$$

where

$$M_1 = \frac{1}{8\gamma^2} \left[-1 + \frac{\sin(\gamma) \cos(\gamma) + \cosh(\gamma) \sinh(\gamma)}{2\gamma(\cosh^2(\gamma) - \sin^2(\gamma))} \right]$$

$$M_2 = \frac{1}{8\gamma^2} \left[\frac{\sin(\gamma) \cos(\gamma) - \cosh(\gamma) \sinh(\gamma)}{2\gamma(\cosh^2(\gamma) - \sin^2(\gamma))} \right] \text{ and } \gamma = \frac{1}{2\sqrt{E}}$$

The dimensionless parameters that appear in the system of equations are defined by:

$$R_a = \frac{\beta \Omega^2 (T_2^* - T_1^*) \Delta R^4}{\nu \kappa} \varepsilon^2$$

$$R_m = \frac{(\rho_p \Phi_r^* + (1 - \Phi_r^*) \rho) \Omega^2 \Delta R^4}{\mu \kappa} \varepsilon^2$$

$$R_n = \frac{(\rho_p - \rho) \Omega^2 \Phi_r^* \Delta R^4}{\mu \kappa} \varepsilon^2$$

$$E = \frac{\nu}{\Omega \varepsilon^2}, \quad L_e = \frac{\kappa}{D_B}, \quad Pr = \frac{\nu}{\kappa}$$

$$N_A = \frac{D_T (T_2^* - T_1^*)}{D_B T_1^* \Phi_r^*}, \quad N_B = \frac{(\rho C)_p \Phi_r^*}{(\rho C)}$$

4. Results and discussion

To examine the effects of different parameters on the onset of convection, we use the shooting method to transform the system of equations 5-7 into a set of six first order differential equations. The solution of this boundary value problem is sought as a superposition of six linearly independent solutions. Each independent solution which satisfies the boundary condition at $r=0$, is constructed by Runge-Kutta numerical scheme of fourth order. A linear combination of these solutions, satisfying the other boundary conditions at $r=1$, leads to a homogeneous algebraic system of the combination coefficients. A necessary condition to obtain non trivial solutions is the vanishing of the determinant, which defines a characteristic equation.

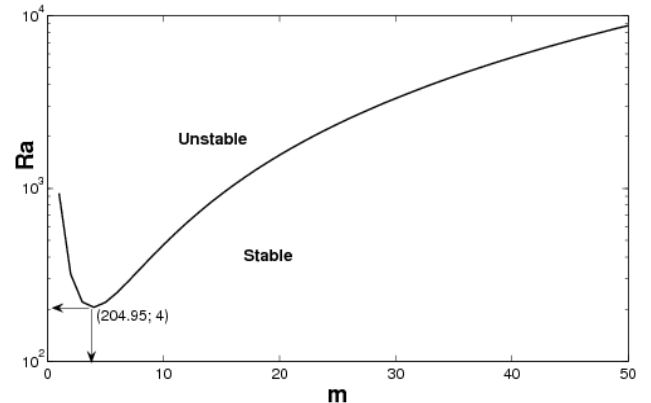


Figure 2: Stability diagram: neutral stability curves for $N_A=2, N_B=0.01, R_n=1, L_e=10, \delta=0.5$ and $E=10^{-3}$.

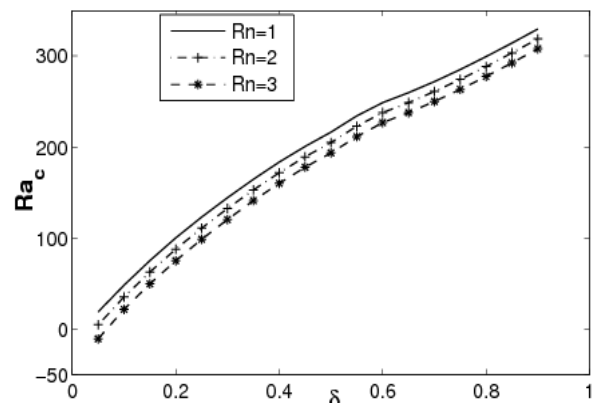


Figure 3: Variation of the critical Rayleigh number with curvature parameter for different values of nanoparticle concentration Rayleigh number with $N_A=2, N_B=0.01, L_e=10$ and $E=10^{-3}$.

An example of neutral stability curves corresponding to the evolution of Rayleigh number, Ra , versus the wave number, m , is presented in figure 2. From this type of curve, we can deduce the corresponding critical Rayleigh number Ra_c and wave number m_c for each dimensionless parameters.

To analyze the effect of nanoparticle volume fraction on the onset of convection, we present in figure 3 the evolution of critical Rayleigh number versus the curvature parameter for different values of the nanoparticle concentration Rayleigh number Rn . It turns out that when Rn increases, the critical thermal Rayleigh number decreases and then destabilizing effect of the nanoparticle volume fraction on the onset of stationary convection. This destabilizing effect is caused by the increasing effects of the Brownian motion and thermophoresis diffusion which increase by increasing the nanoparticle volume fraction.

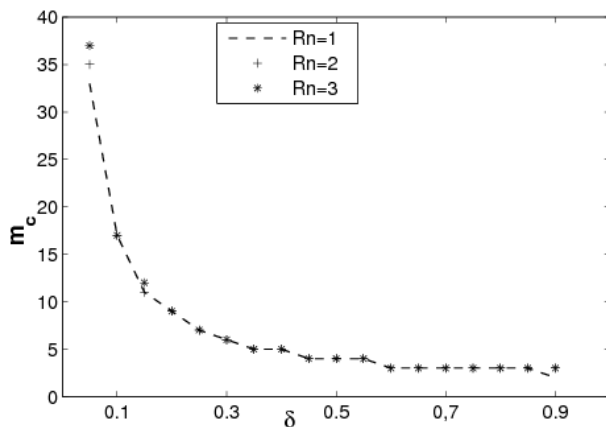


Figure 4: Variation of the critical wave number with curvature parameter for different values of nanoparticle concentration Rayleigh number with $N_A=2$, $N_B=0.01$, $Le=10$ and $E=10^{-3}$.

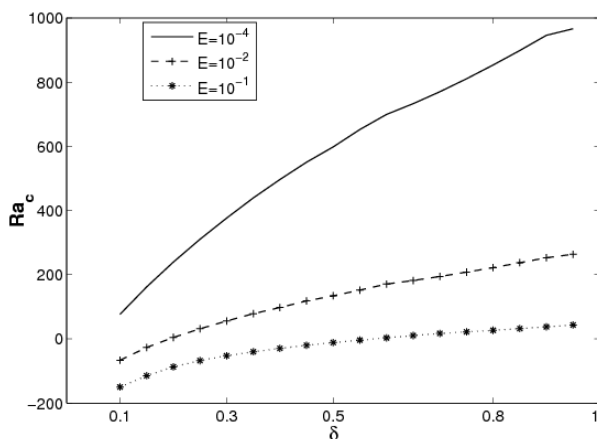


Figure 5: Variation of the critical Rayleigh number with curvature parameter for different values of Eckman number with $N_A=2$, $N_B=0.01$, $Rn=1$ and $Le=10$.

Figure 4 shows that the nanoparticle volume fraction has no effect on the size of the convection cells. The size of the convection cells increases only with the curvature parameter of the Hele-Shaw cell. Finally, the effect of the Eckman number (Coriolis forces) on the onset of

nanofluids convection is illustrated in figure 5. From this figure, we notice that the critical thermal Rayleigh number increases with the inverse of Eckman number and then the Coriolis forces have a stabilizing effect, on the onset of nanofluids convection, which increases by increasing the curvature parameter

1. Conclusion

In this study, we have investigated the onset of nanofluids convection driven by centrifugal force in a rotating annular Hele-Shaw cell using a linear stability theory. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. We have considered that, at the inner and outer cylinders, the nanoparticle volume fraction alters itself together with the consequence of Brownian motion and thermophoresis diffusion. The effects of curvature parameter, Eckman number, nanoparticle concentration Rayleigh number, on the onset of convection have been examined. All results are presented in the form of marginal stability curves depicting the evolution of the critical thermal Rayleigh number and critical wave number. The results show that except Eckman number, which has a stabilizing effect, all others parameters accelerate the onset of nanofluid convection.

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