

RESOLUTION OF THE ELASTICITY PROBLEM WITH MIXED FINITE ELEMENT

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Abstract.

In this paper, we introduce a Mixed Finite Element of elasticity linear problem in two space dimension. This problem is appointed by Lamé system with Neumann and Dirichlet boundary conditions, physically called forces of traction. In the context, we show the existence and uniqueness of the solution of the weak formulation associated with the proposed problem. Then we prove that the discrete condition inf – sup is shown by using the finite element P1-Bubble.P1.

Key words: Linear elasticity, Mixed Finite Elements, existence and uniqueness solution

AMS subject classification:

- mechanical Engineering
- Mathematics applied

Introduction

This paper presents a comprehensive study of Lam's system with Neumann and Dirichlet boundary conditions :

$$\left\{ \begin{array}{l} f = -\mu\Delta U - (\lambda + \mu)\nabla(\nabla.U) \text{ in } \Omega \\ f = (f_1, f_2), U = (u_1, u_2) \\ U = \vec{h} \text{ on } \Gamma_{2,4} = \Gamma_2 \cup \Gamma_4 \\ \mu \frac{\partial U}{\partial n} + \lambda \nabla.U n = \vec{g} \text{ on } \Gamma_{1,3} = \Gamma_1 \cup \Gamma_3 \end{array} \right. \quad (0.1)$$

with U is the displacement, f is the exterior force applied into the domain Ω and λ, μ are the physically constants of the materials of Ω , g is the force of traction.

we prove the existence and uniqueness of weak solution of Lam's system, basing on assumptions mentioned in the article [1].

this later reference had presented all cases that we can meet .

The existence and uniqueness of the solutions to the system are shown under some standard conditions, that

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we will mention later.

To calculate this weak solution we use the discretization by mixed finite element method using P1-bubble.P1. Model building Solid object is deformed under the action of forces applied.

A point in the solid ,originaly in (x, y) , after sometime it will come into (X, Y) , the vector $U = (u_1, u_2) = (X - x, Y - y)$ is called displacement.

When the movement is small and the solid is elastic then HOOK's law gives a relationship between the stress tensor and the strain tensor.

$$\sigma = \lambda \text{tr}(\varepsilon)I_2 + 2\mu\varepsilon \quad (0.2)$$

with : σ is the stress tensor .

$\varepsilon = \frac{1}{2}(\nabla U + (\nabla U)^T)$ is the strain tensor.

I_2 is the identity matrix.

μ is the shear modulus (or rigidity).

where λ is Lam's first parameter.

Navier's equation is given by

$$\text{div}\sigma = \lambda \text{div}(\text{tr}(\varepsilon)I_2) + 2\mu \text{div}\varepsilon \quad (0.3)$$

Then we have :

$$\text{div}\sigma = \lambda \text{div}(\text{tr}(\varepsilon)I_2) + \mu \text{div}(\nabla U) + \mu \text{div}(\nabla U)^T \quad (0.4)$$

with a simple calculation,we find that

$$\text{div}(\text{tr}(\varepsilon)I_2) = \text{div}(\nabla U)^T = \nabla(\text{div}(U)) \quad (0.5)$$

We obtain

$$\rho a = \mu \Delta U + (\lambda + \mu) \nabla(\text{div}U). \quad (0.6)$$

If the solid is in dynamic equilibrium so: $\rho a + f = 0$, f are the external forces applied to the solid .Finally,we result the equation

$$f = -\mu \Delta U - (\lambda + \mu) \nabla(\text{div}U) \quad (0.7)$$

The second term $(\lambda + \mu) \nabla(\text{div}U)$ simulates a preservation volume .
the traction force is given by :

$$\sigma.n = \mu \frac{\partial U}{\partial n} + \lambda(\nabla.U)n \quad (0.8)$$

where n is the unit outward normal.In the above, σ is the Cauchy stress .

In the present investigation, we consider small strains and displacements.

The kinematics equations therefore consist of the strain-displacement relation :

$$\varepsilon = \varepsilon(U) = \nabla_s U. \quad (0.9)$$

The weak formulation :Find $(u, \chi) \in V(\Omega) \times L_0^2(\Omega)$ such that

$$\left\{ \begin{array}{l} \int_{\Omega} \mu \nabla u : \nabla v d\Omega + \int_{\Gamma_{1,3}} \lambda (\nabla \cdot u) n \cdot v d\Gamma_{1,3} + \int_{\Omega} (\lambda + \mu) \chi \nabla \cdot v d\Omega - \int_{\Gamma_{1,3}} (\lambda + \mu) \chi n \cdot v d\Gamma_{1,3} \\ = \int_{\Gamma_{1,3}} g \cdot v d\Gamma_{1,3} + \int_{\Omega} f \cdot v d\Omega \quad \forall v \in V_0(\Omega) \\ \int_{\Omega} (\lambda + \mu) \chi q d\Omega = \int_{\Omega} (\lambda + \mu) q \nabla \cdot u d\Omega \quad ; \forall q \in L_0^2(\Omega) \\ \forall (v, q) \in V_0(\Omega) \times L_0^2(\Omega) \end{array} \right. \quad (0.10)$$

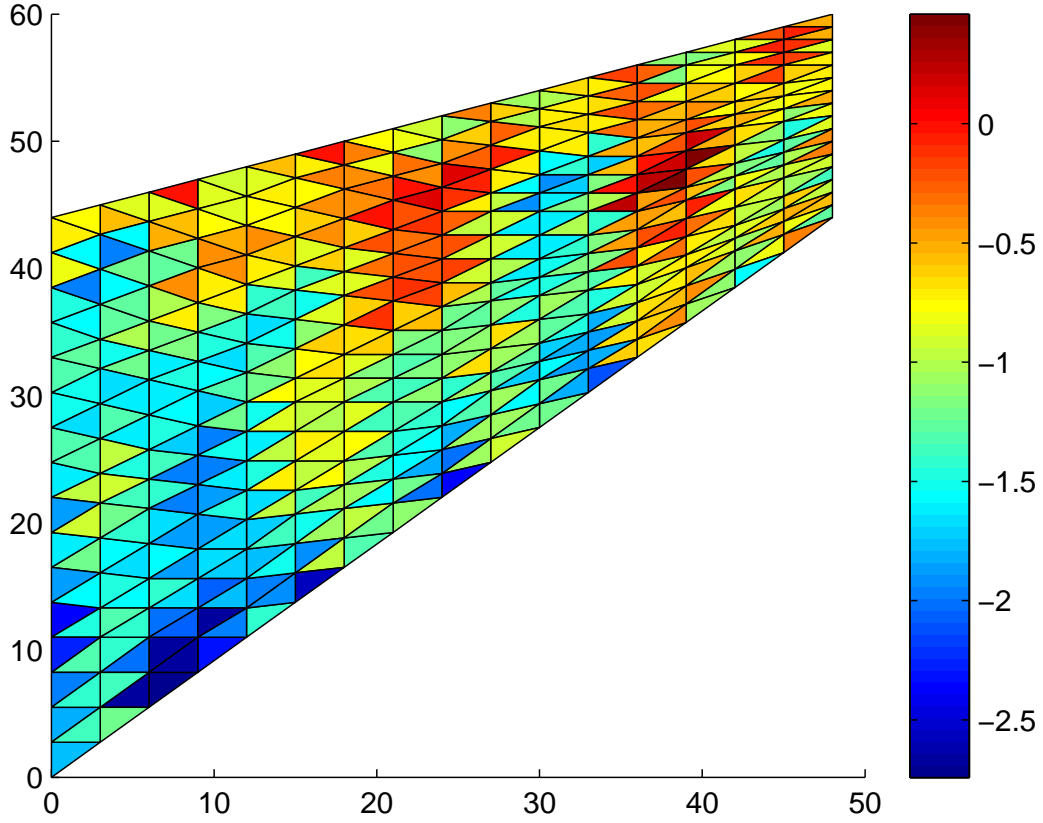


Fig 1:visualization of u_1 of the solution U

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