NATURAL CONVECTION HEAT TRANSFER ENHANCEMENT IN A SQUARE CAVITY PERIODICALLY COOLED FROM ABOVE

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Abstract

The present paper reports numerical results of natural convection within an air filled square cavity with its horizontal walls submitted to different heating models. The temperature of the bottom horizontal surface (hot temperature) is maintained constant, while that of the opposite surface (cold temperature) is varied sinusoidally with time. The parameters governing the problem are the amplitude ($0 \leq a \leq 0.8$) and the period ($\tau \geq 0.001$) of the variable temperature, the Rayleigh number ($10^3 \leq Ra \leq 7\times10^6$) and the Prandtl number ($Pr = 0.71$). In constant cooling conditions ($a = 0$), up to three different solutions (monocellular flow **MF**, bicellular vertical flow **BVF** and bicellular horizontal flow **BHF**) are obtained. Their existence ranges are delineated and the transitions observed are identified and described. In the variable cooling conditions, the effect of the amplitude and the period of the exciting temperature on heat transfer are examined in the case of the **MF** and **BHF**. In comparison with the constant heating conditions, it is found that the variable cooling temperature could lead to a resonance phenomenon characterized by an important increase in heat transfer by about 46.1 %.

Key words: *Natural convection – Periodic cooling – Enhancement of heat transfer*

1. Introduction

Natural convection flows generated by buoyancy forces in rectangular closed cavities have been the object of numerous studies in the past. Actually, considerable efforts are still devoted in the area where this basic geometry remains attractive. In some practical applications involving natural convection as a removal heat transfer mechanism, the energy provided to the system is variable in time and gives rise to unsteady natural convection flow. In addition, thermal and dynamical behaviors of a fluid subjected to time dependent thermal conditions are impossible to predict on the basis of the results obtained with constant temperature conditions due to the strong non linear aspect of the governing equations. Consequently, the thermal exchanges within the considered system could be improved or reduced compared to the stationary regime. Hence, many studies have been performed with oscillating temperature boundary conditions [1-3].

The majority of the published works consider cavities with the driving heating walls customarily held at a constant temperature or heat flux or varying in a sinusoidal manner with time. However, no attempt has considered the inverse case where the cavities are heated with a constant temperature and submitted to a time dependent cooling. This intermittent cooling generates convection in an unsteady manner and a good understanding of free convective flows under such thermal boundary conditions is necessary to avoid overheating phenomena which are frequently encountered in practical situations. Hence, the objective of the present study consists to investigate the effect of the periodic cooling temperature to the cooling process inside the cavity. The effect of the parameters related to the variable temperature (amplitude a and period τ) on the dynamical and thermal behaviors is examined.

2. Problem formulation

A schematic of the physical problem and the associated boundary conditions is shown in Fig. 1. It consists of a two-dimensional square enclosure with adiabatic vertical walls and thermally active horizontal walls. Specifically, the horizontal upper wall of the enclosure is cooled with a temperature varying sinusoidally with time while the horizontal lower wall is heated at a constant temperature. The non-dimensional governing equations, written in the vorticity stream-function formulation (Ω-Ψ), are :

$$\frac{\partial \Omega}{\partial t}+\frac{\partial (u\Omega)}{\partial x}+\frac{\partial (v\Omega)}{\partial y}=\Pr\left[\frac{\partial^2 \Omega}{\partial x^2}+\frac{\partial^2 \Omega}{\partial y^2}\right]-\mathrm{Ra}\,\Pr\frac{\partial T}{\partial x} \qquad (1)$$

$$\frac{\partial T}{\partial t}+\frac{\partial (uT)}{\partial x}+\frac{\partial (vT)}{\partial y}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2} \qquad (2)$$

$$\frac{\partial^2 \Psi}{\partial x^2}+\frac{\partial^2 \Psi}{\partial y^2}=-\Omega \qquad (3)$$

2.1. Boundary conditions

The hydrodynamic boundary conditions are such that the stream function and the velocity components are zero on the rigid walls of the cavity. The thermal boundary conditions associated to the governing equations are:

$T = 1$ for $y = 0$ and $0 \le x \le 1$

$T = a \sin(2\pi t/\tau)$ for $y = 1$ and $0 \le x \le 1$

$\frac{\partial T}{\partial x} = 0$ for $x = 0, 1$ and $0 \le y \le 1$

2.2. Heat transfer

At each time step, the mean Nusselt number on the cooled wall is evaluated as :

$$\mathrm{Nu}(t)=\int_0^1 \left.\frac{\partial T}{\partial y}\right|_{y=1} dx \qquad (4)$$

The mean Nusselt number during one flow period, τ_C, is evaluated as:

$$\overline{\mathrm{Nu}}=\frac{1}{\tau_C}\int_0^{\tau_C}\mathrm{Nu}(t)dt \qquad (5)$$

3. Results and discussion

In the following, the results obtained are first presented in terms of Nu variations with Ra for all the solutions obtained (case of constant heating conditions) then, in the variable regime, time averaged value of the Nusselt number are presented.

3.1 Constant active temperatures *(a = 0)*

The preliminary tests conducted show that the multiplicity of solutions is strongly dependent on the Rayleigh number and the initial conditions imposed to the system. The streamlines corresponding to the three modes of convection are reported in Fig. 2. By using the conduction regime to initiate the numerical code for Ra = 10^5, the final steady state is a dominating positive (or negative) monocellular flow (**MF**) with two small negative (or positive) vortices located in the vicinity of two opposite corners of the cavity (Ψ_{max} = 26.04 and Ψ_{min} = –0.12). A progressive increase of Ra was found to be favorable to the secondary flow (to the small vortices). In fact, for Ra = 2.5×10^6 (value marking the threshold above which the monocellular flow transits towards another type of flow), Ψ_{max} = 119.66 and Ψ_{min} = –15.21 which indicates that the increase rate of the small cells with Ra is more important than that of the main flow. Now, by decreasing Ra below 10^5 and using initial conditions favorable to the monocellular flow, the latter is maintained until the transition towards the conductive regime which occurs at Ra ≈ 2.5×10^3 (Ψ_{max} = 0.01846, Ψ_{min} = –0.00001 and Nu = 1.00003).

The second kind of solution obtained is the bicellular horizontal flow (**BHF**) which is characterized by the formation of two horizontal cells with comparable intensities. The obtaining of this kind of solution was facilitated by the transition of the **MF** observed at Ra = 2.6×10^6; this transition occurs towards the **BHF**. Once obtained, the **BHF** was used to start the numerical code and its existence in the steady regime was delineated by varying Ra in the range [3.5×10^5, 6.8×10^6]. Below this range, the solution obtained is monocellular even if the initialization of the numerical code is in favor of the **BHF**. Above this range, it was verified that, for Ra = 7×10^6, the **BHF** is unsteady periodic.

The last kind of solution obtained for this configuration is the bicellular symmetrical vertical flow (**BVF**) which is a bicellular ascending flow, presenting a symmetry with respect to the vertical plane x = 0.5. In the steady state regime, this solution was maintained for Ra varying in the range [1.1×10^4, 4.1×10^4]. Below this range of Ra, the transition occurs directly towards the **MF.** However, a window of periodic oscillatory **BVF** (**OS BVF**) was observed in the range [4.2×10^4, 5.2×10^4] and, above this range, the transition occurs directly towards the **MF** as it was verified for Ra = 5.3×10^4. The arrows in Fig. 2 indicate the transition undergone by each solution when Ra is progressively increased (blue arrows) or decreased (red arrows).

The variations of the Nusselt number versus the Rayleigh number are presented in Fig. 2 for all the solutions obtained. In this figure, it is seen that the existence of the **BHF** and the **BVF** are restricted respectively to high and relatively intermediate Rayleigh numbers while the **MF** is obtained in a wide range of this parameter. In addition, in the steady regime, the **MF** is seen to be the most favorable to the heat transfer as it permits a better thermal interaction between the horizontal active boundaries.

3.2 Variable temperature ($a \neq 0$ and $\tau \neq 0$)

A set of preliminary tests shows that the time-averaged values of the Nusselt number $\overline{\text{Nu}}$, obtained for various periods, are seen to be different from those corresponding to the case of a constant cooling temperature. These differences in favor of the variable cooling temperature let suppose the existence of the resonance phenomenon. To illustrate this resonance phenomenon, we present in Fig. 3 the variations of $\overline{\text{Nu}}$ versus the period τ in the case of the **BHF** for Ra = 10^6 and various values of the amplitude. In Fig. 3, we note the important increase of $\overline{\text{Nu}}$ engendered by the increase of the period τ in the case of moderate amplitudes even for small values of a. The maximum values of $\overline{\text{Nu}}$ are obtained for a critical value $\tau_{cr} \approx 0.01$ independent of the amplitude a. Further increase of τ above τ_{cr} leads to a continuous decrease of $\overline{\text{Nu}}$. Hence, the importance of the resonance phenomenon is favored by increasing the amplitude of the cooling temperature and the range of τ for which $\overline{\text{Nu}}$ is higher than the Nusselt number corresponding to $a = 0$ (constant cooling condition) is enlarged by increasing a.

4. Conclusion

Natural convection in a square cavity, heated from below with a constant temperature and cooled from above with a time periodic temperature, was studied numerically. The results obtained in the steady regime show the existence of three types of solutions. The existence of the resonance phenomenon is proved in the present study, and it is found to be more intensified by increasing the amplitude of the exciting temperature. The system enters in resonance with the exciting temperature for a critical period which is independent of the amplitude. For this critical period, the results reported show that the periodic cooling can be used to enhance notably the heat losses in comparison with the case of constant cooling temperature.

References

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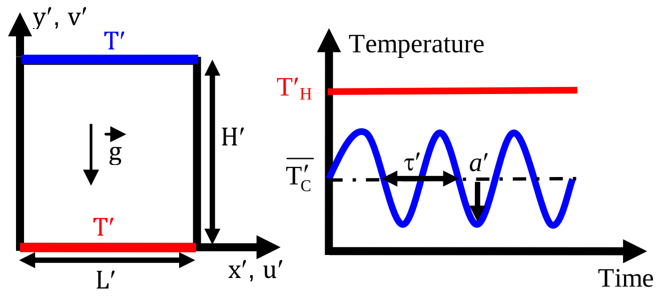

(a) (b)

Figure 1: a) Geometry of the problem and b) Thermal excitations

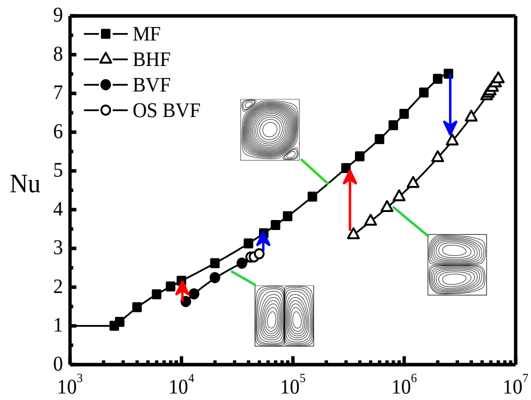

Figure 2: Variations of the Nusselt number versus the Rayleigh number for all the solutions obtained in the steady state regime.

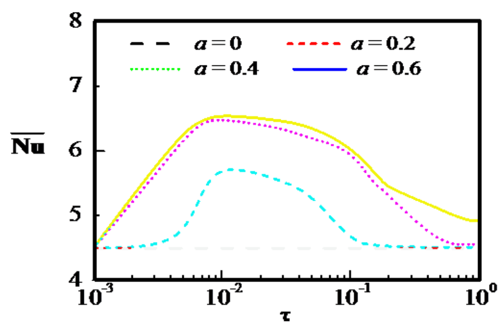

Figure 3: Effect of the period τ on the time averaged Nusselt number for Ra = 10^6 and various values of a in the case of a bicellular flow structure.