Cracked Surface Prediction of Composite Plates Subjected to Low Velocity Impact

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Abstract

In this investigation, the fiber-reinforced composite plates subjected to low velocity impact are studied by the use of finite element analysis (FE). To predict matrix cracking, the dynamic stress analysis is carried out by the use of a constitutive equation of composite laminates without damage. The impact induces damage at higher impact velocity including matrix cracking is predicted by the appropriate failure criteria. The present results indicate that matrix cracking appears in the upper 90° plies with the dominance of transverse shear stress. The shape and dimensions of the cracked surfaces of different layers of a composite plate are numerically simulated. The cracked surface of the first layer increases in the time. At time $t = 30.51\mu s$, the cracked surfaces of the first two and last two layers are similar in the shape and dimensions**.** It is revealed also that at time $t = 66{,}69\mu s$ elements located far from the contact point are also affected by cracking. These elements tend to disappear starting from the first layer at the last layer.

Keywords*:* Finite element analysis, composite plates, impact, failure criteria**,** matrix cracking

1. Introduction

In the present study, a Mindlin's plate theory with a modified Hertzian contact law is used to describe the behavior of Carbon/Epoxy laminated composite plates subjected to a central impact of a spherical rigid projectile. We intend, in the present work, to predict impact damage at a specific impact speed. Thus, we proceed for the computation of the dynamic stresses at the Gaussian points via our numerical model, then failure criterion suggested by Choi and Chang [1] are used to predict matrix cracking and delamination.

2. Finite element formulation

The displacements *u*, *v* and *w* at any point (*x*, *y*, *z*) in the laminate are given by:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

Where u_0, v_0 and w_0 denote the mid-plane displacements and θ_x and θ_y denote the rotations along the *x* and *y* axes, respectively.

3. Contact force

The dynamic equation of plate is given by equation (2):

$$[M]\{\ddot{u}\} + [K]\{u\} = \{F\} \tag{2}$$

The dynamic equation of a rigid ball is given by the use of the Newton's second law:

$$m_i \ddot{w}_i = -F_c \tag{3}$$

Where m_i is the mass of the ball (impactor) and F_c is the contact force.

The contact force between the impactor and the plate is calculated using a modified non-linear Hertzian indentation law proposed by Tan and Sun [2-4]:

For the loading phase the contact force is given by:

$$(F_c)_{n+1} = K\left[q - (w_s)_{n+1} - \beta_1 (F_c)_{n+1}\right]^{1.5} \tag{4}$$

For the unloading phase:

$$(F_c)_{n+1} = K_1\left[q - (w_s)_{n+1} - \alpha_0 - \beta_1 (F_c)_{n+1}\right]^{q} \tag{5}$$

Where,

$$q = (w_i)_n + \Delta t(\dot{w}_i)_n + \left(\frac{\Delta t}{4}\right)(\ddot{w}_i)_n \,,\ \beta_1 = \frac{\Delta t^2}{4m_i} \quad k_1 = \frac{F_m}{(\alpha_m - \alpha_0)^{2.5}}$$

4. Prediction of impact induced damage

Once the numerical model has been validated, we intend to predict impact damage at a specific impact speed. Thus, we proceed for the computation of the stresses at the Gaussian points via our numerical model, then failure criterion suggested by Choi and Chang [1] are used to predict matrix cracking and delamination. The critical matrix cracking criterion is given by the following equation:

$$\left(\frac{{}^n\bar{\sigma}_{22}}{{}^nY}\right)^2 + \left(\frac{{}^n\bar{\tau}_{23}}{{}^nS_T}\right)^2 = e_M^2$$

$$if \quad e_M \geq 1 \quad failure \tag{6}$$

$$if \quad \bar{\sigma}_{22} \geq 0 \quad {}^nY = {}^nY_t \ \ and\ if \quad \bar{\sigma}_{22} < 0 \quad {}^nY = {}^nY_c$$

Where Y_T and Y_c are the in situ ply transverse tensile and compressive strengths, respectively. $\bar{\tau}_{23}$ and $\bar{\sigma}_{22}$ are the averaged interlaminar transverse shear stress, and normal stress respectively [5,6].

The delamination criterion is given by the equation bellow:

$$D_a\left[\left(\frac{{}^{n}\bar{\tau}_{23}}{{}^{n}S_T}\right)^2+\left(\frac{{}^{n+1}\bar{\tau}_{13}}{{}^{n+1}S_L}\right)^2+\left(\frac{{}^{n+1}\bar{\sigma}_{22}}{{}^{n+1}Y}\right)^2\right]=e_D^2 \quad (7)$$
$$if \quad e_D \geq 1 \quad failure$$
$$if \quad \bar{\sigma}_{22} \geq 0 \quad {}^{n+1}Y={}^{n+1}Y_T$$
$$if \quad \bar{\sigma}_{22} < 0 \quad {}^{n+1}Y={}^{n+1}Y_c$$

Where D_a is a constant to be determineted from the experiment. *n* and *n*+1 correspond to upper and lower plies of the *n*th interface, respectively .

5. Damage analysis

5.1 Prediction of matrix cracking

To predict matrix cracking, dynamic stresses are calculated at nine Gaussian points of each element. The failure criteria given by equation (6) is then applied at each Gaussian point. Only $\sigma_{22}\, and\, \tau_{23}$ stress components are concerned in the prediction of matrix cracking.

The material properties of the impactor and composite plate and the yielding laminate strength are given in the table below.

Table 1: Material properties of plate and impactor and yielding strength of lamina

Composite materials	
Plate properties	
$E_1 = 144.8\,GPa$, $E_2 = 9.65\,GPa$	
$G_{12} = G_{13} = 7.10\,GPa, G_{23} = 5.92\,GPa$	
$\nu_{12} = 0.30, \rho = 1389.2\,Kgm^{-3}$	
Boundary conditions	*Simply-supported*
Lamina Strengths	
$Y_T = 55.20$ MPa ,	$D_a = 1.8 - 2.0$
$Y_C = 294.0$ MPa ,	$S_T = 32.94$ MPa
$S_L = 101.1$ MPa	
Impactor properties	
$R = 6.35mm, E = 200GPa, \nu = 0.3$	
$\rho = 7870\ Kgm^{-3}$	

The stacking sequences $[0_2/90_2/0_2]$ is considered. The plate is simply supported on all edges. Its size is $127\text{mm x } 76.2\text{mm}$ and $t_{layer} = 0.465\,mm$. The time step used is $\Delta t = 0.27\ \mu s$. If we use 15x11 elements the impact force is applied on the up surface at the 83th element in the Gaussian point (ipgs=2 and jpgs=2) with $(NPGS = 3)$).

In this following section, we intend to determine matrix cracking initiation using an impact velocity equal to $12\ m/s$. Table 2 shows that matrix cracking first appear in element 83 (ipgs=1 and jpgs=3) of lamina 3 with transverse Shear stress dominates $(\bar{\tau}_{23} = 33.73MPa, \bar{\sigma}_{22} = -3.62MPa)$. The next failure is observed in lamina 4 at the same time. In lamina 6 matrix cracking appear at around $12.42\,\mu s$. It has to be pointed out that the stiffness matrix is not modified to take into account of matrix cracking.

Table 2: Position of matrix cracking, $[0_2/90_2/0_2]$.

Lam. Nu	**Ele. Nu**	**IPGS**	**JPGS**	$t(\mu s)$	$\bar{\sigma}_{22}$ (MPa)	$\bar{\tau}_{23}$ (MPa)
3	83	1	3	11.34	-3.62	33. 73
3	83	2	3	11.34	-3.62	-33.73
4	83	1	3	11.34	-1.21	33.73
4	83	2	3	11.34	-1.21	-33.73
6	83	3	1	12.42	13.74	-32.52
6	83	3	2	12.42	13.74	32.52

5.2 Prediction of cracked surface

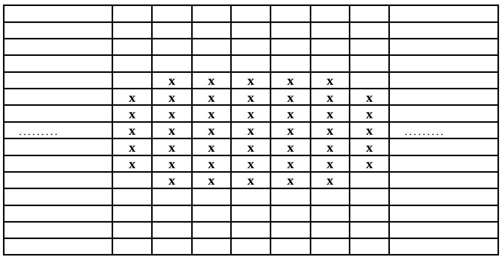
Numerical simulation of the shape and dimensions of the cracked surfaces of different layers of a composite plate, of stacking $(0_2/90_6/0_2)$ and dimensions $127x76.2\ mm^2$, subjected to an impact at a velocity equal to $20m/s$ and $R = 12mm$ has shown in figure (1). The stresses are computed at each Gaussian point. In this figure, we find that the cracked surface of the first lamina, subjected to the impact, increases by a value $S_1 = 1161{,}288\,mm^2$ at $t_1 = 30.51\mu s$ to a value $S_2 = 1935{,}48\,mm^2$ at $t_2 = 58.04999\mu s$ then it goes to a value of $S_3 = 2399{,}9952\,mm^2$ at $t_3 = 66.69\mu s$. The figure plotted shows that at $t_1 = 30.51\mu s$ the cracked surface is the same for the considered plies. This surface equal to $S_1 = 1161{,}288\,mm^2$. This surface at $t_2 = 58.05\mu s$ decreases to a value $S_2 = 1935{,}48\,mm^2$ in the first lamina to a value $S_2 = 1780{,}6416\,mm^2$ in the second lamina then increases to a value $S_2 = 1833{,}8672\,mm^2$ in the fifth lamina and finally it passes to a value $S_2 = 1780{,}6416\,mm^2$ in laminas 9 and 10. At $t_3 = 66.69\mu s$ the surface decreases to a value $S_3 = 2399{,}9952\,mm^2$ in the first lamina to a value

$S_3 = 20387{,}7056\,mm^2$ in the second lamina then to a value $S_3 = 18838{,}8672\,mm^2$ in the fifth lamina and finally to a value $S_3 = 17800{,}6416\,mm^2$ in laminas 9 and 10. In ply 10 (figures 2), the cracked surfaces are less important and from an impact over time $t = 58\mu s$, the damage stabilizes.

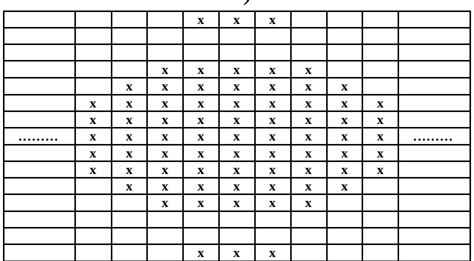
6. Conclusion

In the present work, low velocity impact induces damage in laminated composite plates is numerically studied. A finite element approach based on a 9 node Lagrangian element including the effect of transverse shear stress is used. A parametric study has been conducted prior to damage prediction. The tendency of the results show that the matrix cracking appears in the upper plies with the dominance of transverse shear stress τ_{23} and in the lower plies with the dominance of both transverse shear stress τ_{23} and normal stress σ_{22} which explains the presence of two types of matrix cracking due to bending and to shear. It is also demonstrated from a dynamic stress analysis that crack initiation occurs away from the contact point.

a)



b)



c)

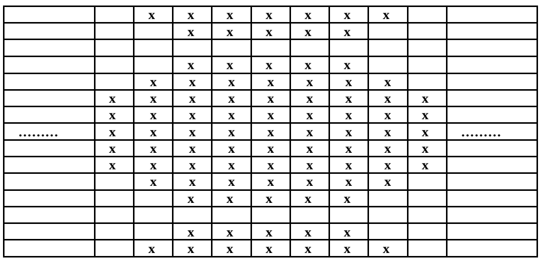
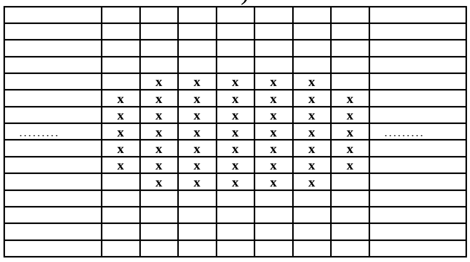


Figure 1. Prediction of the matrix cracking at different times of the first lamina.
a) $t_1 = 30.51\mu s$. b) $t_2 = 58.05\mu s$. c) $t_3 = 66.69\mu s$.

a)



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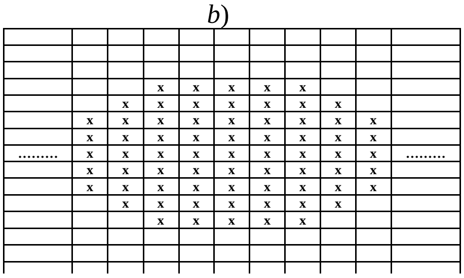

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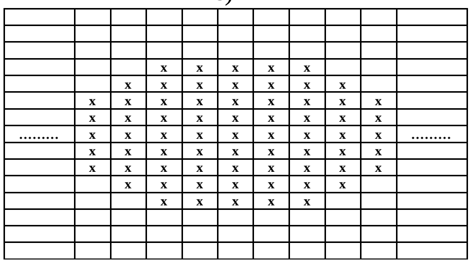

Figure 2. Prediction of the matrix cracking at different times of the lamina 10.
a) $t_1 = 30.51\mu s$. b) $t_2 = 58.05\mu s$. c) $t_3 = 66.69\mu s$.

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