Evaluation de la distance effectivede la zone d'élaboration en mécanique de la rupture par la triaxialité des contraintes

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ABSTRACT: *The distribution of stress triaxiality is determined in the vicinity of notch defect performed in the specimen submitted to compression load. The calculations are carried out by using 3D finite element analysis. The evaluation of notch stress intensity factor is made by using the volumetric approach. A new method to evaluate the effective distance in this approach is proposed.*

Keywords –*Stress triaxiality, notch defect, stress intensity factor, volumetric approach and effective distance.*

I. INTRODUCTION

In fracture mechanics, the critical stress intensity factor or so called fracture toughness and other mechanical proprieties are measured in some conditions of geometry, loading mode and constraint. So, they have to be applied in the same conditions. The transferability of these proprieties is imperative in structural design because change of geometry or constraint conditions may promote brittle fracture. This transferability is based on the stress triaxiality which takes into account the tridimensional stress field. In this paper, β is used as a measure of stress triaxiality. This parameter is defined as the ratio of the hydrostatic stress over the equivalent Von Mises stress:

$$\beta = \frac{\sigma_h}{\sigma_{eq,VM}} \quad (1)$$

Where:
$$\sigma_h = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}$$

And:

$$\sigma_{eq,VM} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_2)^2}$$

It is well known that ductile fracture is sensitive to the stress triaxiality. There are several indicators to quantify the state of constraints at defect tip in the literature. Over the list of these indicators, one can quote the T-Stress [1], the Q parameter [2], the multiaxiality parameter [3] and the *p* parameter [4].

The evaluation of notch stress intensity factor is made by using the volumetric approach which supposes that the fracture process requires a certain fracture volume. This volume is assumed as a cylinder with effective distance at its diameter. In this paper, we propose a new method based on stress triaxiality to evaluate the effective distance.

II. MATERIAL

The material studied is high-strength steel 45CrMoSi6 according to the French standard. Its mechanical and chemical proprieties are listed in Table I and Table II.

Table I

Chemical proprieties of 45CrMoSi6

C	Mn	Si	Cr	Mo
0.45	0.6	1.6	0.6	0.25

Table II

Mechanical proprieties of 45CrMoSi6

E (GPa)	ν	σ_Y (MPa)	σ_U (MPa)	A%	Density (kg/m^3)
210	0.28	1463	1662	2.8	7800

III. GEOMETRICAL CHARACTERISTICS

Tests were performed using U-notched circular specimens (Fig.1) with the external radius Re=20 mm, internal radius Ri=10 mm, thickness B=7 mm, and the notch length a=4 mm. Different notch radii were obtained using a wire-cutting electrical discharge machine (EDM) [5]. Four notch radii are considered: ρ = 0.3, ρ = 0.5, ρ = 1.0 and ρ = 2.0 mm. The specimens are submitted to compression load in order to determine the critical loads when the fracture occurs. These loads are introducing to the simulation computation to finally evaluate the critical notch stress intensity factor and the stress triaxiality evolution.

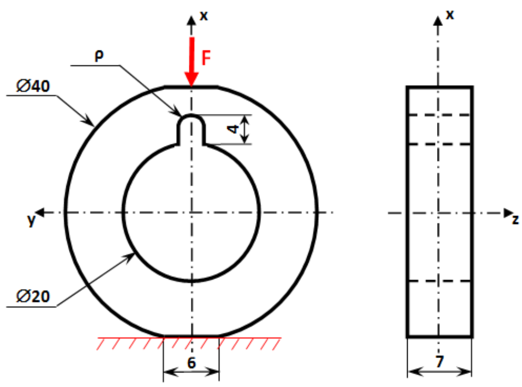

Fig.1. Specimen geometry and compression load.

IV. NOTCH STRESS INTENSITY FACTOR AND VOLUMETRIC METHOD

Stress distributions around the notch defect have been converted into so called notch stress intensity factor using the notch fracture mechanics and particularly the volumetric method.

The volumetric method [6] is a local fracture criterion, which supposes that the fracture process requires a certain fracture volume. This volume is assumed as a cylinder with effective distance at its diameter.

The elastic-plastic stress distribution along the ligament is plotted in the bi-logarithmic diagram as can be seen in Fig.2:

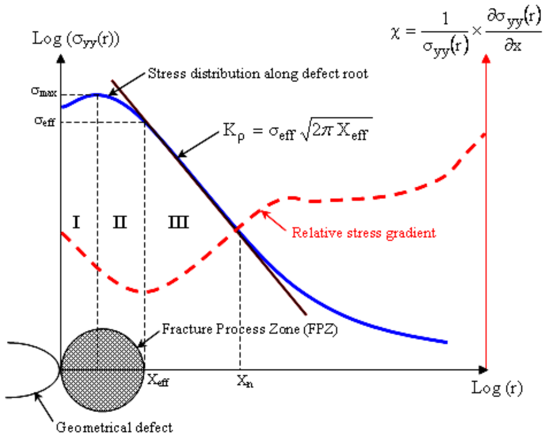

Fig.2. Schematic elastic-plastic stress distribution along notch ligament and stress intensity concept.

The notch stress intensity factor [N.S.I.F] is defined as function of effective distance and effective stress:

$$K_{\rho} = \sigma_{eff} \sqrt{2\pi X_{eff}} \quad (2)$$

The effective distance corresponds to the minimum of the relative stress gradient which given as:

$$\chi(r) = \frac{1}{\sigma_{yy}(r)} \frac{\partial \sigma_{yy}(r)}{\partial r} \quad (3)$$

The effective stress is considered as the average volume of the stress distribution over the effective distance. However stresses are multiplied by a weight function in order to take into account the influence of stress gradient due to geometry and loading mode. The effective stress is defined as:

$$\sigma_{eff} = \frac{1}{X_{eff}} \int_0^{X_{eff}} \sigma_{yy}(r) \times (1 - r \times \chi(r))\, dr \quad (4)$$

V. FINITE ELEMENT ANALYSIS

The part modelled in 3D analysis is meshed by quadrangular elements with eight nodes. Computing was carried out on Castem software 2014.

By using the maximum hoop stress as failure criterion [7], the stress distribution was simulated front of the notch (Fig.3) in the opening fracture mode.

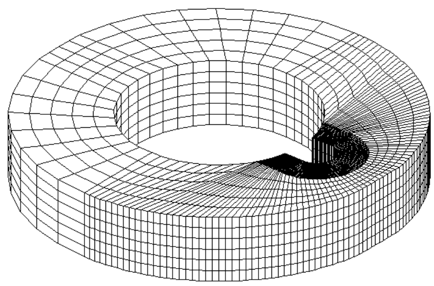

Fig.3.Mesh density in specimen.

The results of numerical simulations exhibit that for all notch radii, the hoop stress $\sigma_{\theta\theta}$ which is maximal at the middle of the notch, is predominant (Fig.5). Then, we will be interested in the radial direction in the middle of specimen where the crack is expected to occur first [8]and grow up radially (Fig.4).

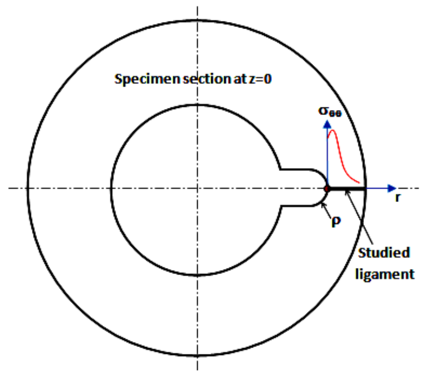

Fig.4.Studied ligament in the notch tip of specimen.

The stress evolutions are plotted versus the notch tip distance for each radius (Fig.5). The maximum stress values decrease when the notch radius increases.

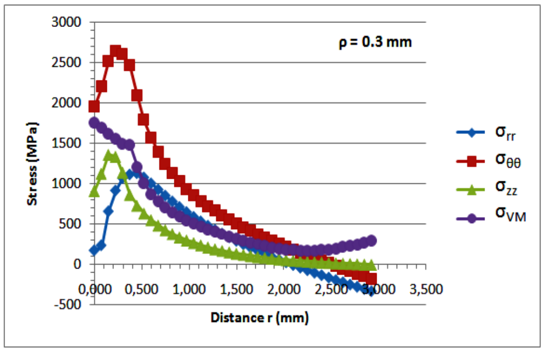

Fig.5.Stress distributions in the notch tip ligament.

VI. RESULTS

The critical notch stress intensity factor (C.N.S.I.F.) noted K_{ρ}^{c} is computed by using the critical loads leading to the fracture specimens [5]. The critical effective distance X_{eff}^{c} and the associated critical effective stress σ_{eff}^{c} are given by numerical simulations. These two characteristics are taken into account to determine the C.N.S.I.F. $K_{\rho}^{c}$by using the relation (2). The results are summarized in table III.

Table III

Critical Notch Stress Intensity Factor for various notch radii

Notch radius ρ (mm)	Critical effective stress σ_{eff}^{c} (MPa)	Critical effective distance X_{eff}^{c} (mm)	Critical notch stress intensity factor K_{ρ}^{c} (MPa.m$^{0.5}$)
0.30	1961.670	0.375	95.221
0.50	1895.018	0.450	100.765
1.00	1744.737	0.600	107.126
2.00	1782.460	0.825	128.333

The evolution of the relative stress gradient along the notch tip ligament is given in Fig.6. The critical effective distance X_{eff}^{c} corresponds to the minimum of relative stress gradient, as mentioned in section IV. All effective distances and associated effective stresses are reported above, in table III.

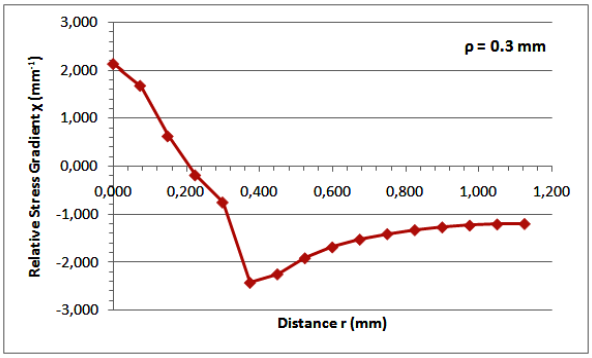

Fig.6.Relative stress gradient distribution in the notch tip ligament.

The stress triaxiality evolutions along the ligament are shown in Fig.7. These curves present two convex forms with an intersection in a point (M). It is proven that this intersection point is at a distance d_M on the ligament which correspondsto the effective distance X_{eff}^{c} (Table IV). One can conclude that the stress triaxiality can be used to determine the effective distance as that can be made with the relative stress gradient. The effective distance is then:

$$X_{eff}^{c} = d_M \tag{5}$$

Where the point M is located at a distance d_M and defined as:

$$\begin{cases} \lim_{r \to d_M +} \frac{d\beta}{dr} > 0 \\ \lim_{r \to d_M -} \frac{d\beta}{dr} < 0 \end{cases} \tag{6}$$

Table IV

Comparison of distance d_M with effective distance X_{eff}^{c}.

Angle ρ (mm)	effective distance X_{eff}^{c} (mm)	distance d_M(mm)
0.30	0.375	0.375
0.50	0.450	0.450
1.00	0.600	0.600
2.00	0.825	0.875

According to the figure below, we can also note that the amplitude of the critical maximum of triaxiality decreases when the notch radius increases.

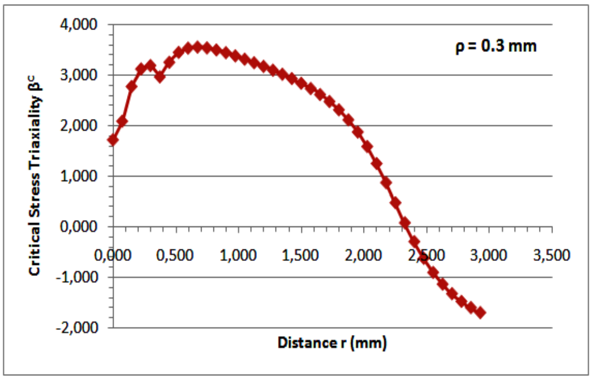

Fig.7. Critical stress triaxiality evolutions in the notch tip ligament.

VII. CONCLUSION

The volumetric approach requires the determination of the diameter of fracture elaboration zone and the associated stress in the bilogarithmic diagram to evaluate the notch stress intensity factor. That is made by numerical computation of the effective distance X_{eff}^{c} and effective stress σ_{eff}^{c}. The maximum tangential stress is used as fracture criterion. In order the take into account the tridimensional stresses; the evolution of stress trixiality β is studied along the notch tip ligament. The stress triaxialitycurves reveal two convex forms whose point of intersection is located at a distance noted d_M which proved to be equal to the effective distanceX_{eff}^{c}. One can conclude that stress triaxiality can be used to determine the diameter of fracture elaboration zone in volumetric approach. In other words, the value of the effective distance can be obtained from the stress triaxiality is equal to d_M.

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