Multi-phase differential scheme modeling of voided viscopiezoelectric composite materials

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Abstract:
In this paper, the effective behavior of multi-phase voided viscopiezoelectric composite materials is investigated based on the combination of the Laplace transform and multi-phase differential scheme modeling.
The multi-phase differential scheme model is developed and applied to multiphase multifunctional composites. A procedure of constructing gradually the behavior of the composites by adding a series of small volume fractions of the reinforcement to the initial material (matrix) is developed. The prediction of the effective behavior is done in the frequency domain then an inversion to the time one is performed numerically. Different numerical results are presented and a comparison with the predictions of the Mori-Tanaka mean field micromechanical model is done. The effect of different parameters on the effective behavior is shown.

Keywords: Effective behavior, viscopiezoelectric, composite materials, Carson transform, Mori-Tanaka, Multiphase Differential scheme.

1. Introduction

Piezoelectric materials are widely used in many industrial fields due to the electroelastic coupling effect (piezoelectric effect) exhibited by these materials. Ceramic piezoelectric materials are brittle and susceptible to fracture. The addition of a polymer phase give them more ductility and formability and also an improvement of the hydrostatic coefficient compared to the one presented by the piezoelectric ceramic phase. Also, researchers have shown that composites with tailored porosity exhibit properties and futures that cannot be achieved by dense ones. Due to the above presented features of this kind of materials, the development of a micromechanical modeling predicting accurately the behavior of these materials while taking into account the porosity and the time dependent behavior of the polymer phase is of key interest in order to design piezoelectric composites suited for special industrial application.
In this work, the effective behavior of multi-phase voided viscopiezoelectric composites is modeled based on the correspondence principle describing the linear time dependent behavior of viscopiezoelectric materials and on a multi-phase differential scheme model. The use of the Carson transform allows the extension of the multi-phase differential scheme micromechanical model to the Carson domain. The effective properties are first predicted in the frequency domain then a numerical inversion is performed to the time one. A comparison with the prediction of the Mori-Tanaka model is presented. The effect of different parameters on the behavior of the considered composite is shown. Special focus is given to effective coefficients of practical interest.

2. Constitutive equations for linear viscopiezoelectric materials

This work deals with the viscopiezoelectric behavior of piezoelectric composites with a polymeric matrix. These kinds of composites show time dependent properties. The time dependent constitutive model that describes the linear viscopiezoelectric behavior of piezoelectric homogeneous material is obtained by generalizing the Blotzmann principle of linear viscoelastic materials. One can write [3]:

$$\begin{aligned} \sigma_{ij}(t) &= \int_0^t c_{ijkl}(t-\tau)\frac{d\varepsilon_{kl}(\tau)}{d\tau}d\tau + \int_0^t e_{lij}(t-\tau)\frac{dE_l(\tau)}{d\tau}d\tau \\ D_i(t) &= \int_0^t e_{ikl}(t-\tau)\frac{d\varepsilon_{kl}(\tau)}{d\tau}d\tau + \int_0^t \kappa_{il}(t-\tau)\frac{dE_l(\tau)}{d\tau}d\tau \end{aligned} \tag{1}$$

Using the condensed notation [3], the time dependent constitutive model takes the following form:

$$\Sigma_{ij}(t) = \int_0^t E_{iJKl}(t-\tau)\frac{dZ_{Kl}(\tau)}{d\tau}d\tau \tag{2}$$

where
$$E_{iJKl}(t) = \begin{cases} c_{ijkl}(t) & (J,K=1,2,3) \\ e_{lij}(t) & (J=1,2,3;K=4) \\ e_{ikl}(t) & (J=4;K=1,2,3) \\ -\kappa_{il}(t) & (J=4;K=4) \end{cases}$$
is the time dependent viscopiezoelectric relaxation tensor, Σ and Z are the generalized stress and strain respectively.
The convolution integral (2) gives a mathematical model that relates the generalized stress Σ to the generalized strain Z through the viscopiezoelectric relaxation tensor $E(t)$.
The Carson transform of a time dependent function is given by:

$$f(s) = s\int_0^t f(t)e^{-ts}dt \tag{3}$$

The application of the Carson transform on Eq. (2), transforms the viscopiezoelectric problem to an equivalent piezoelectric one in the frequency domain. One can write the constitutive equation in the Carson domain as follow

$$\Sigma_{iJ}(s) = E_{iJKl}(s) Z_{Kl}(s) \quad (4)$$

Equation (4) shows the analogy between the linear viscopiezoelectric problem and linear piezoelectric one.

3. Multi-phase differential scheme modeling

The differential scheme is based on the idea that the behavior of composites with dilute concentration of volume fraction of inclusions could be predicted accurately. The construction of the effective behavior is done gradually by adding a series of finite volume fractions of inclusions to the initial material (matrix) [7]. This process is illustrated here with differential equations.

Let us consider that we are at the jth stage of the incremental process and the considered multi-phase viscopiezoelectric composite is constituted of a matrix and N inclusions. The effective properties can be written as follows

$$L^{eff}_{iJKl}(c_1, c_2, ..., c_N, s) = L^m_{iJKl}(s) + \sum_{I=1}^{N} c_I (L^I{}_{iJMn}(s) - L^m{}_{iJMn}(s)) A^{I(Dil)}_{MnKl}(L^m(s)) \quad (5)$$

Where $c_1, c_2, ..., c_N$ are respectively the volume fraction of inclusions, s is a complex variable, $L^m_{iJKl}(s)$ is the relaxation tensor of the matrix et $L^I_{iJKl}(s)$ is the relaxation tensor of the Ith inclusion.

The dependence of L^{eff} on the volume fractions ($c_1, c_2, ..., c_N$) and $A^{I(Dil)}$ on L^m is explicitly indicated.

As described in [7] at the stage (j+1)th, the made up composite at the jth stage is considered as homogeneous medium of the effective properties $L^{eff}(c_1, c_2, ..., c_N, s)$ and a small volume ΔV of the homogeneous medium is removed, and the removed volume is replaced with ΔV_1 of material 1, ΔV_2 of material 2 ,….and ΔV_N of material N.

The effective properties at this stage are derived by:

$$L^{eff}_{iJKl}(c_1 + \Delta c_1, c_2 + \Delta c_2 + ... + c_N + \Delta c_N, s) = L^{eff}_{iJKl}(c_1, .., c_N, s) + \sum_{I=1}^{N} \frac{\Delta V_I}{V_0} (L^I{}_{iJMn} - L^{eff}_{iJMn}(c_1, .., c_N, s)) A^{I(Dil)}{}_{MnKl}(L^{eff}(c_1, .., c_N, s)) \quad (6)$$

The last equation is transformed to [7]:

$$\Delta L^{eff}_{iJKl}(s) = \sum_{I=1}^{N} (\Delta c_I + c_I \frac{\Delta c}{1-c})(L^I{}_{iJMn}(s) - L^{eff}_{iJMn}(s)) A^{I(Dil)}{}_{MnKl}(L^{eff}(s)) \quad (7)$$

with $\Delta L^{eff}(s) = L^{eff}(c_1 + \Delta c_1, c_2 + \Delta c_2, ..., c_N + \Delta c_N, s) - L^{eff}(c_1, .., c_N, s)$

Introducing fractional constants ($p_1, p_2, ..., p_N$) for the respective inclusion materials (1, 2,..,N) in such a way that $c_1 = p_1 c,\ c_2 = p_2 c, ..., c_N = p_N c$, the relationship between the volume concentration of inclusions is given by:

$$c = p_1 c + p_2 c + ... + p_N c \quad (8)$$

This implies that

$$\Delta c = p_1 \Delta c + p_2 \Delta c + ... + p_N \Delta c$$

and

$$p_1 + p_2 + ... + p_N = 1 \quad (9)$$

Based on this equation, Eq. (7) leads to

$$\Delta L^{eff}_{iJKl}(s) = \Delta c \sum_{I=1}^{N} (p_I + p_I \frac{c}{1-c})(L^I{}_{iJMn}(s) - L^{eff}_{iJMn}(s)) A^{I(Dil)}{}_{MnKl}(L^{eff}(s)) \quad (10)$$

When Δc becomes infinitesimal, Eq. (10) can be rewritten in the following generalized ordinary differential form

$$\frac{dL^{eff}_{iJKl}}{dc} = \frac{1}{1-c} \sum_{I=1}^{N} p_I (L^I{}_{iJMn} - L^{eff}_{iJMn}) A^{I(Dil)}_{MnKl}(L^{eff}) \quad (11)$$

with the initial condition $L^{eff}(c=0) = L^m$. $L^m(s)$ represents the moduli of the matrix and c is the sum of the volume fractions of all inclusions that reinforce the matrix.

Eq. (11) allows the prediction of the effective behavior of multi-phase viscopiezoelectric composites in frequency domain while taking into account the volume fractions and shape of inclusions. The time dependent effective behavior is obtained by numerically inverting Eq. (11) to the time domain.

4. The multi-phase Mori-Tanaka mean field

The Mori-Tanaka mean field approach is widely used in the prediction of composites behavior. The main assumption of the Mori-Tanaka mean field approach is the consideration that the infinite reference medium has the properties of the matrix. So, one can write the expression of the concentration tensor as follow [4]:

$$A^{MT}_{SjMn}(s) = A^I_{KlSj}(s)(f_m I_{KlMn} + \sum_{I=2}^{N} f_I A^I_{MnKl}(s))^{-1} \quad (12)$$

with

$$A^I_{MnKl}(s) = (I_{KlMn} + \frac{1}{V^I} \bar{T}^{II}_{iJKl}(E^m(s)) \Delta E^I{}_{iJMn}(s))^{-1}$$

The effective behavior of multi-phase viscopiezoelectric composites, based on the Mori-Tanaka model, is obtained through the following equation

$$L^{eff}_{iJKl}(s) = L^m_{iJKl}(s) + \sum_{I=1}^{N} c_I (L^I{}_{iJMn}(s) - L^m{}_{iJMn}(s)) A^{I(MT)}_{MnKl}(s) \quad (13)$$

The Mori-Tanaka model is presented for the sake of comparison with the predictions of the developed differential scheme.

5. Numerical results

In this section, numerical results for three-phase voided viscopiezoelectric composites are presented. The considered composite is constituted of polymer matrix (LaRC-SI) reinforced by piezoelectric ($BaTiO_3$) and voided inclusions. The used properties are given in [4].

Note that all the coefficients of the tensor E^{eff}_{iJKl} are computed in frequency and time domains based on the differential scheme and the Mori-Tanaka model. Only e_{33} is numerically presented here based on the Mori-Tanaka model. The storage and loss part of the effective piezoelectric e_{33} moduli are presented for the considered composites with respect to the frequency. One can see that the effective piezoelectric coefficients are time dependent even that the piezoelectric phase is considered to be time independent. Also it is seen that when the percentage of void is increased, the piezoelectric coefficients of the composite become more prominent.

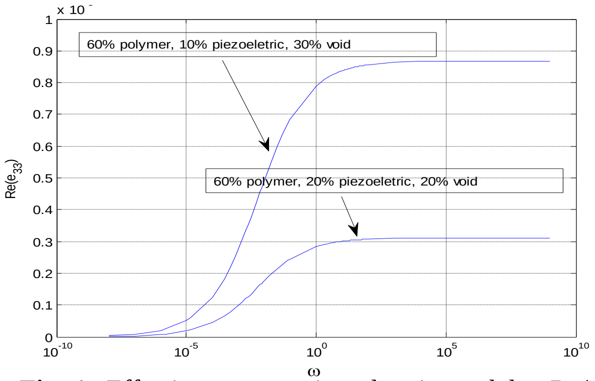

Fig. 1: Effective storage piezoelectric modulus $Re(e_{33}(\omega))$ for spherical viscopiezoelectric composite constituted of polymer matrix reinforced by voided and piezoelectric inclusions.

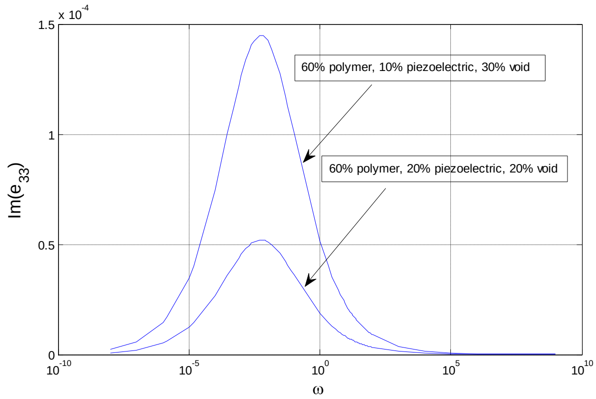

Fig. 2: Effective loss piezoelectric modulus $Im(e_{33}(\omega))$ for spherical viscopiezoelectric composite constituted of polymer matrix reinforced by voided and piezoelectric inclusions.

6. Conclusion

In this work, the multi-phase differential scheme is developed and used for the prediction of the effective behavior of multi-phase voided viscopiezoelectric composites. The combination of the correspondence principle and the multi-phase differential scheme model is elaborated. The effective behavior is predicted in the frequency domain then inverted numerically to the time one. The multiphase Mori-Tanaka model is also presented. In frequency domain, the storage and loss part of all predicted effective piezoelectric moduli can analyzed. Numerical results are presented for a voided viscopiezoelectric composite constituted with polymer phase and voided and piezoelectric inclusions. Even that piezoelectric phase is considered to be time independent**,** it is shown that piezoelectric coefficients of the reinforced composite are time dependent which means that the polymer phase affects the behavior of the composite. Also it is shown that the piezoelectric effect is more prominent when the percentage of void is increased in the composite. The developed modeling allows the design of voided viscopiezoelectric composites with optimized features.

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