# On the generalization of contour integral in three-dimensionalcrack problem

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## Abstract
This paper present a new analytical formulation for three-dimensional crack growth problem. This work is based on a generalization of the Rice's integral for three-dimensional crack problem.A new integral parameter in real three-dimensional case, which computes the energy release rate combining an arbitrary crack front, is developed. A physical interpretation allows to evaluate the efficiency of the proposed integral. The new integral is compared with the local path independent integral of Bui[2] in three-dimensional cases.
The non-dependent domain is justified by numerical approach. The energy release rate distribution along the crack front line is obtained, and compared to Bui's integral [2].A numerical validation, in terms of energy release rate, is carried out on a DCB specimen under opening mode loading for wood material.

**Keywords:** *Crack problem, DCB specimen, Energy release rate,finite element, J-integral,Three-dimensional,Wood material,*

## 1. Introduction

The complex mechanical loading and high climatic variations on structures implies having a better understanding of their fracture mechanical behavior. It is necessary to consider three-dimensional case in the study of reel crack growth problem.For it the most common approach is the determination of the energy release rate by energetic approach.
Most of the studies carried out deal with two-dimensional case. The studies of wood structures request the development of specific tools for three-dimensional configurations.
The topic of this paper deals with the generalisation of the J-integral formalism to a three-dimensional problem and its adaptability of the G-theta method for a finite element implementation.

## 2. Analytical formulation

For plan problem and for static crack Rice (1968) [1] has defined A path independent integral which allows to computes energy release rate around the crack tip. For cracked linear elastic material, Rice [1] have used $J$-integral to compute energy release rate for curvilinear contour. J-integral takes the following notation:

$$J^{2D} = \int_{\Gamma} \left(W.n_1 - (\sigma_{ij}.n_j.u_{i,1})\right).d\Gamma$$

Where $W$denotes the strain energy density, $\Gamma$ is arbitrary curvilinear contour oriented by its normal vector, $u_i$ is the displacement component and $\sigma_{ij}$ is the stress component.

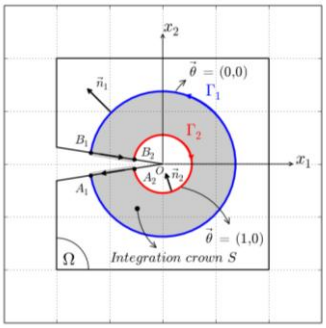


Fig. 1: Description integral domain of$J$ and $G_\theta$

For three-dimensional problem, the crack front line is defined as the intersection of two surface (figure 2).

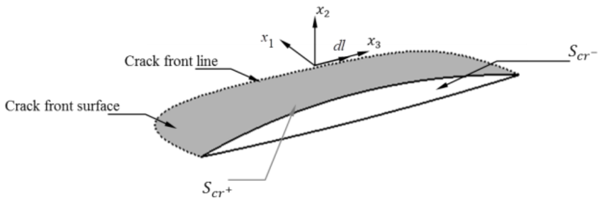


Fig. 2: Crack surface with crack system coordinate
To take into account in global formalism the most general case which the crack front line is arbitrary, we consider some volume and surfaces integration domains(figure 3).

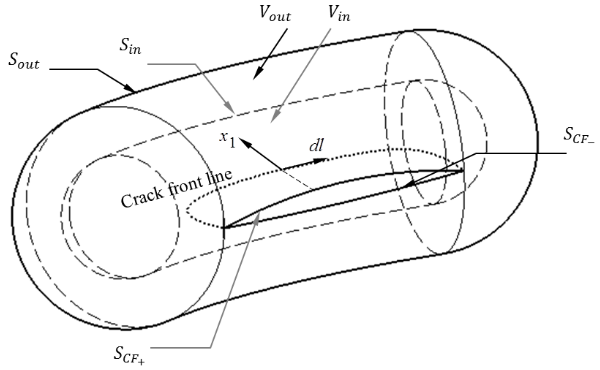


Fig. 3: Closed volume and surfaces integration domains

The new $J^{3D}$-integral formulation is based on energetic approach using consideration described in figure 3. After some mathematical developments, we obtain:

$$J^{3D} = \int_{S_{out}} \left(W.n_k - (\sigma_{ij}.n_j.u_{i,k})\right).dS - \int_{S_{CF}} \sigma_{ij}.n_j.u_{i,k}.dS + \int_{V_{out}} \left(\sigma_{ij}.(\varepsilon_{ij})_{,k} - W_{,k}\right).dV$$

the $J^{3D}$-integral is composed by three separated terms. The first one designates the classical part used for the determination of the crack growth initiation. It can be completed by the effects of a crack lips pressure introduced by the second term. The last part allows the generalization for the crack propagation ensuring the non-path dependence when the crack tip moves inside the integral domain.The $J^{3D}$ is interpreted as the integration of the $J^{3D}_{Bui}$-integral along the crack front line:

$$J^{3D} = \lim_{A(\Gamma)\to 0}\left(\int_{cfl} J^{3D}_{Bui}.dl\right)$$

$J^{3D}_{Bui}$ denotes Bui integral for three-dimensional crack problem. The $J^{3D}$-integral can be used for the evaluation of the average value of the energy release rate $\tilde{G}$ along the crack front line:

$$\tilde{G} = \frac{J^{3D}}{\int_{cfl} dl}$$

To implement this integral in a finite element software, it is easier to consider a volume domain integral [3, 4].

$$G^{3D}_{\theta} = -\int_{V} \left(W.\theta_{k,k} - (\sigma_{ij}.u_{i,k}).\theta_{k,j}\right).dS - \int_{S_{CF_+}+S_{CF_-}} \sigma_{ij}.u_{i,k}.n_j.\theta_k.dS - \int_{V_{in}} \left(W_{,k} - \sigma_{ij}.(\varepsilon_{ij})_{,k}\right).\theta_k.dV$$

The $G^{3D}_{\theta}$ allows to compute the distribution of energy release rate along the crack front line per slim thickness. We can establish a relation between $G^{3D}_{\theta}$ and $J^{3D}_{Bui}$ as follows:

$$J^{3D}_{Bui} = G^{3D}_{\theta}(\omega)$$

The average energy release rate per every slim thickness $G^{3D}_{\theta}(\omega)$ can be equal to energy release rate per each plan $J^{3D}_{Bui}$ in studied solid.

## 3. Numerical validation

The numerical implementation is based on aDouble Cantilever Beam (DCB)loaded in an open mode. The DCB specimen was adapted by Dubois et al to wood material. In this part, we recall the wood specimen dimensions of DCB device. Fig. 4 presents the dimensions in millimetres of the initial wood specimen. In this wood specimen, two holes are machined in order to fixe the Arcan device. This allows a loading fixations in tensile mode. The geometry of the DCB specimen has been optimized by using a finite element computation. This specimen is adapted to obtain a stable crack growth rate during propagation for tensile mode.

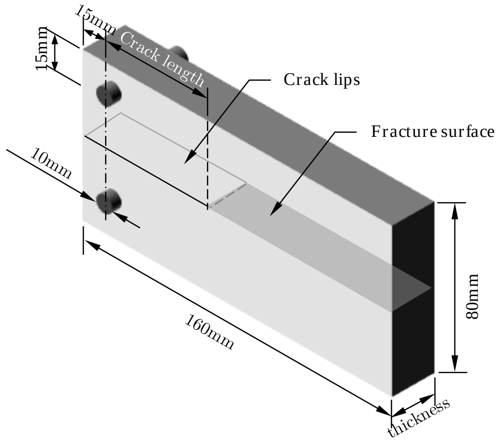


Fig. 4: DCB specimen

The finite element computation is realized for an elastic isotropic behavior. Wood material used is Douglas fire and has the following elastic characteristics E=14100 MPa, Poisson ratio NU=0.3.The initial crack length is fixed to 60mm.

In thefollows, the results of numerical study are exposed. In order to observe the effect of thicknesses on the DCB specimenwe plot the evolution of the energy release rate as function of the crack front line.

The description of $\theta$ field around the crack front lineis shown inFig. 4. The $\theta$ field is equal to zero on outside surface, and 1 on inside surface.

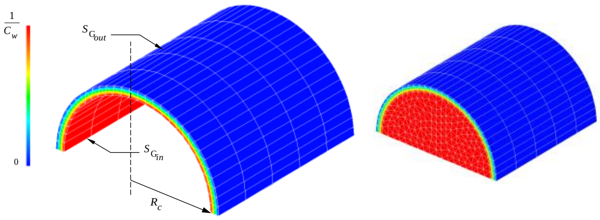

Fig. 5: Description of $\theta$fieldaround the crack front line

Let us analyse now the influence of the thickness on the energy release rate. As shown in Fig. 6, the support of the theta field is supported by a cylindrical plate

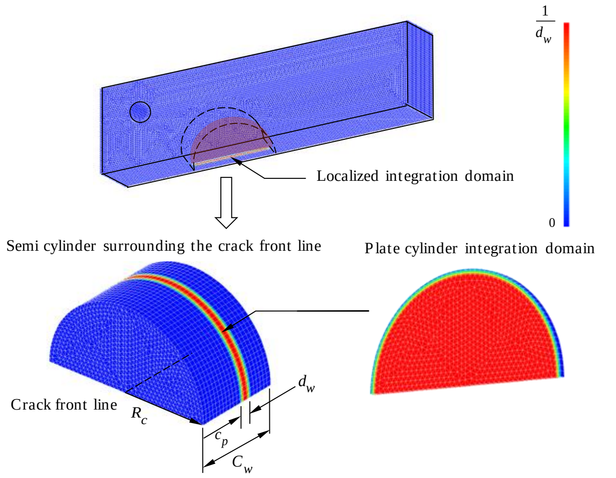


Fig. 6: Integral cylindrical plate domain around the crack tip

The average energy release rate is calculated along $d_w$ (Fig. 6). Fig. 7 shows us the energy release rate distribution along the crack front line versus the thickness.

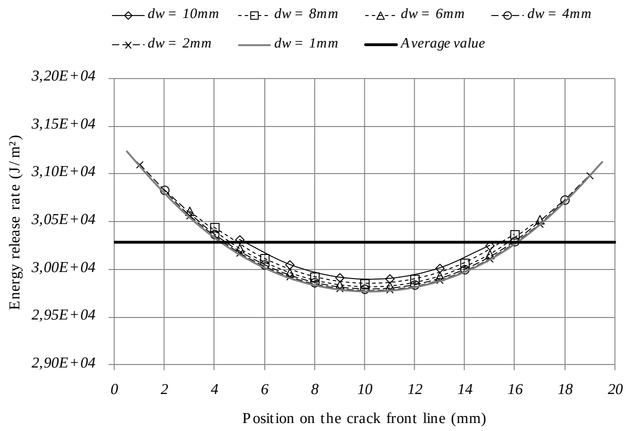


Fig. 7: Energy release rate distribution along the crack front line

Numerical approach allowed us to evaluate the distribution of energy release rate thanks to $G_\theta^{3D}$-integral in tensile mode.

## 4. Conclusion et perspectives

This paper deals with a new formulation of the J-integral for the study of fracture process in element by taking into account three dimensional effects. A theoretical and numerical approach are established.

At this stapes, more numerical investigations are necessary.Also, it will be necessary to extend the $J$ integral to a mixed mode loading [5] case for three dimensional problems.

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