Elastic-perfectly plastic analysis of coupled soil-structure system with local uplift

Nadia El Abbas[1], Khamlichi Abdellatif[2] and Bezzazi Mohammed[1]

1. Department of Physics, Faculty of Science and Techniques Tangier, nadia.elabbas@gmail.com, bezzazi@hotmail.com
2. Department TITM, ENSA Tetouan, khamlichi7@yahoo.es

Abstract

The objective of this study is to analyze the effects caused by uplift of foundation and soil elastic-perfectly plastic behavior on the dynamic seismic response of a structure. The soil-structure interaction has been modeled by Saint Venant's units mounted in parallel and distributed over the entire width of the foundation.

The model considers the case of small rotation of the base. However, the equations are strongly nonlinear and the obtained system of differential equations is stiff. This was integrated by using the Matlab command ode15s. The response of the structure was studied after that as function of the chosen soil model: elastic or elastic-perfectly plastic. It is shown that the perfectly plastic analysis yield smaller rotational angles and tip displacement when compared with the elastic analysis of the coupled system.

Key Words: *Soil-structure interaction, elastic-perfectly plastic soil, uplift, seism*

1. Introduction

Simplified analytical models for soil-structure interaction were proposed by many researchers. Winkler foundation based model consisting of a mat mounted on distributed vertical springs that describe soil elasticity was one of the first models that has been proposed [1,2]. This model proved to be effective both in terms of accuracy and computational cost for civil engineering structures that are undergoing small lateral excitations.

For more intense lateral loadings, many revised Winkler foundation based models were introduced [3]. An investigation of the seismic response of a gravity rigid pier with shallow foundation on elastic-plastic Winkler foundation with uplift effects was presented in [4]. Seismic performance of a bridge-foundation system was recently considered through discrete mat-foundation system that was coupled to the soil through discrete units [5].

In the present work, the effect on the seismic response system response under 1940 El Centro earthquake record for a structure undergoing foundation local uplift and mounted on elastic perfectly plastic soil type is considered in the small rotation regime. The structure is represented by a single-degree-of freedom system which is bonded on rigid foundation, the soil-structure interaction is modeled by Saint Venant's units that are mounted in parallel and distributed over the entire width of the foundation. Each unit consists of an elastic element which is connected in series with a plastic element. Use of spring elements enabled to depict the initial elastic soil deformation while allowing gradual transition from soil elastic strain range of strains to plastic range of strains.

The discrete coupled soil-structure model takes into account the degrees of freedom related to mat lateral displacement, base vertical displacement and base rotation with this last considered to remain small. The non-linear differential systems of equations of motion of the system are integrated by using the Matlab command ode15s which can handle

2. Materials and Methods

2.1 Modeling of soil structure

The system considered for the structure consists of single-degree-of freedom of mass m , stiffness k and damping c which is supposed to be mounted on a rigid foundation basis. This last is assumed to react as a rigid rectangular plate of negligible thickness and having the mass m_0 taken to be uniformly distributed over its length. The total moment of inertia is designated by I_0 .

The soil-structure interaction takes place at the interface separating the rigid footing and the foundation soil. The coupling of the system is modeled by Saint Venant's units mounted in parallel. Each unit contains an elastic spring connected in series with a plastic element which is activated if the stress from the elastic spring reaches the yield stress σ_y , according to the constitutive behavior shown on figure 1. k_w is the soil modulus of elasticity per unit length. The units are distributed over the entire width of the foundation, see figure 2. Horizontal slippage between the mat and supporting elements is assumed to be negligible.

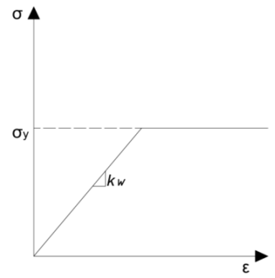


Figure 1. Elastic perfectly plastic behavior of a Saint-Venant's unit

In the elastic range of deformation, the stress-strain relationship is given by: $\sigma = k_w(v \pm x\theta)$ where v is the vertical displacement of the base mat centre of gravity θ is the angle of rotation of the mat base and x is the abscissa along the foundation with origin taken at its centre. When the stress reaches the threshold σ_y for some plastic frontier point x_{pi} , where $i = l \text{ or } r$ that designates left or right edge, then $\sigma = \sigma_y$. Also local uplifting occurs if $\sigma = 0$ for some critical position point x_{ci} .

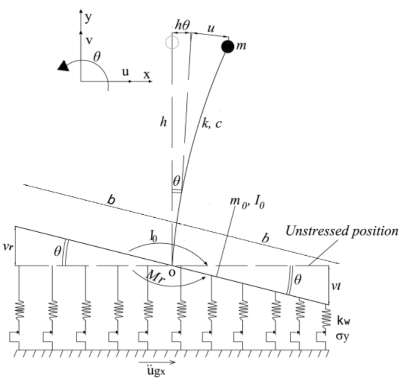


Figure 2. Flexible structure on elasto-perfectly plastic foundation

In figure 2, h designates the height of the structure from the base, M_r the total moment acting on the base mat, u the elastic horizontal displacement of the mat tip relative to the base, $\ddot{u}_{gx}$ the seismic acceleration and b half width of foundation mat. The following notations are also introduced: $\omega = \sqrt{k/m}$ natural frequency of the rigidly supported structure; $\omega_v = \sqrt{k_w/m_0}$ vertical vibration frequency of foundation; $\alpha = h/b$ slenderness ratio; $\beta = \omega_v/\omega$ frequency ratio; $\gamma = m/m_0$ foundation mass to superstructure mass ratio and $\xi = c/(2m\omega)$ damping ratio of the rigidly supported structure.

The elastic perfectly plastic analysis and uplifting nonlinearity are dealt with according to the following steps:

Step 1: under a selected ground motion and for actual positions of base that are associated to uplift or plastic frontier conditions, compute the answer of the coupled system for the given increment and obtain: (i) rotation of the base, (ii) vertical displacement, (iii) horizontal displacement.

Step 1: Compare the actual obtained value of θ with zero.

Step 2: Determine the actual value of critical positions x_{ci} which correspond to $\sigma = 0$.

Step 3: Determine the actual value of plastic positions x_{pi} which correspond to $\sigma_y = k_w\left(v \pm x_{pi}\theta\right)$.

Step 4: Compare x_{pi} with algebraic value of half width of foundation mat.

Step 5: Compare x_{pi} with x_{ci} .

2.2 Equations of motion

In the present study, the rotations are assumed to be enough small, so the following approximations can be made:

$$\cos\left(\frac{u}{h}+\theta\right) \approx 1 \quad \text{and} \quad \sin\left(\frac{u}{h}+\theta\right) \approx \frac{u}{h}+\theta \tag{1}$$

The equations of motion of the entire system of Figure 2, under horizontal acceleration are derived by taking into account the equilibrium of the coupled foundation-mat system. The three equilibrium equations are:

- Equilibrium of forces acting on each degree of freedom in the horizontal direction: $\sum F_x = 0$
- Equilibrium of forces in the vertical direction: $\sum F_y = 0$
- Equilibrium of moment about the center of the foundation of the mat: $\sum M_z = 0$

These equations of motions may be expressed as:

$$m\ddot{u} + mh\ddot{\theta} + c\dot{u} + ku = -m\ddot{u}_{gx}(t) \tag{2}$$

$$\begin{aligned}\ddot{v} = -\frac{k_w}{(m+m_0)}(&vx_{pl} + \frac{1}{2}\varepsilon_2 x_{pl}^2\theta + vb - \frac{1}{2}\varepsilon_1^2\varepsilon_2 b^2\theta + vx_{pr} \\ &+ \varepsilon_2 x_{pr}^2\theta - vx_{pl} - \frac{1}{2}\varepsilon_1^2\varepsilon_2 x_{pl}^2\theta + vb + \varepsilon_2 b^2\theta - vx_{pr} \\ &- \frac{1}{2}\varepsilon_1^2\varepsilon_2 x_{pr}^2\theta) - g\end{aligned} \tag{3}$$

$$\begin{aligned}\ddot{\theta} = \frac{3\alpha^2}{\gamma h}\left(c\dot{u} + ku\right) - \frac{3\varepsilon_2\alpha^2 k_w}{m\gamma h^2}(&\frac{1}{2}vx_{pl}^2 + \frac{1}{3}\theta x_{pl}^3 - \frac{1}{2}\varepsilon_1^2 vb^2 \\ &+ \frac{1}{3}\theta b^3 + \frac{1}{2}vx_{pr}^2 + \frac{1}{3}\theta x_{pr}^2 - \frac{1}{2}\varepsilon_1^2 vx_{pl}^2 - \frac{1}{3}\varepsilon_1^2\theta x_{pl}^3 + \frac{1}{2}vb^2 \\ &+ \frac{1}{3}\theta b^3 - \frac{1}{2}\varepsilon_1^2 vx_{pr}^2 - \frac{1}{3}\varepsilon_1^2\theta x_{pr}^2)\end{aligned} \tag{4}$$

The contact coefficient ε_1 is equal to unity when the contact takes place, and continuously depends on the rotation θ of the foundation and the vertical displacement during the partial uplift, it is expressed as follows:

$$\varepsilon_1 = \begin{cases} 1 & \text{contact at both edges} \\ \varepsilon_2 v / b\theta & \text{left or right edge uplifted} \end{cases} \quad (5)$$

The coefficient ε_2 depends on contact condition of the edges of the base with soil:

$$\varepsilon_2 = \begin{cases} -1 & \text{left edge uplifted} \\ 0 & \text{contact at both edges} \\ 1 & \text{right edge uplifted} \end{cases} \quad (6)$$

The vertical displacement on both edges of the foundation noted and measured from the initial position are:

$$v_i = v \pm b\theta(t),\ i = l, r \quad (7)$$

Because the foundation cannot extend above its initial unstressed position an edge of the foundation mat would uplift when :

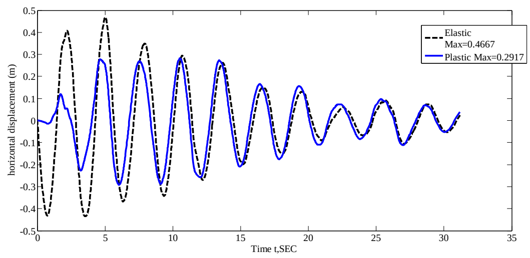
$$v_i > 0,\ i = l, r \quad (8)$$

The equations of motion of the system are formed by equations (2) to (8). This is a highly nonlinear system of second order ordinary differential equations. Its integration can be achieved iteratively by using the Matlab command ode15s which can handle stiff-like systems.

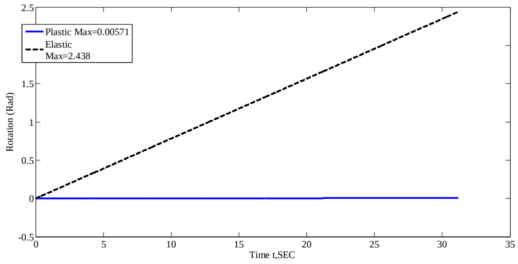
3. Results and discussions

In the following, the systems responses in both ranges elastic and perfectly plastic of the mat-foundation base are considered under a scaled ground motion of the 1940 El Centro earthquake.
The system response is studied with the following set of parameters:

$$h=12m,\ b=12m,\ m=105{,}07.10^3 kg,\ m_0 = 10507 Kg$$
$$\sigma_y = 1.5 Pa,\ k_w = 960\times10^6\, N/m,\ c = 21.85\times10^3\, Kg.s^{-1}$$
$$k = 4.55\times10^5\, N/m,\ \omega=2.08Hz,\ \xi=0.05,\ \alpha=4,\ \beta=8,\ \gamma=0.1.$$



(a)



(6)

(b)

Fig. 3. Response of the structure under El Centro ground motion: (a) horizontal displacement; (b) base rotation for elastic analysis and elastic perfectly plastic analysis

Figures 2 and 3 show that the rotations remain small for the elastic-perfectly plastic type of analysis as plastic strain moderate the magnitudes. On the contrarily during elastic analysis, rotations are great and even rocking is observed.

4. Conclusions

(8)

The effect of local uplifting on a discrete soil-structure interacting system was analyzed. Both elastic and elastic-perfectly plastic soil behaviors were considered. The system was subjected to a scaled 1940 El Centro earthquake record. It was found that in elastic-perfectly plastic analysis the obtained lateral displacement and base rotation were smaller than those obtained for an elastic analysis. The rotations remained small when plastic behavior was integrated. On the opposite, the rotation was great for the elastic analysis and even rocking of the structure has happened.

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