Dynamic Analysis and Motion Control of a Planar Parallel Manipulator

B. ABOULISSANE[1], D. EL HAIEK, L. EL BAKKALI

Modeling and Simulation of Mechanical Systems Team
Abdelmalek Essaadi University, Faculty of Sciences
Tetouan, Morocco
b.aboulissane@gmail.com[1]

Abstract

In this paper, dynamic analysis of a parallel robot is presented using the most popular approach named the Newton-Euler formulation. In this method, the dynamic formulation for each limb and the moving platform must be derived which leads to the calculation of all forces, and this can be helpful in the design of parallel manipulator. In this way, dynamic formulations of the proposed model are used in motion control strategy of this robot to track and analyze its performance.

Keywords: *Parallel manipulators; Newton-Euler formulation; PD controller*

1. Introduction

Dynamic analysis of parallel robots is more complex than the serial robots due to their closed loop structure. However, dynamic modeling is relatively essential for control strategies, where precision and desired dynamic performance are required. Numerous techniques have been used for the dynamic analysis of parallel robot [1][2][3]. These robots have many types of applications. The type we are interested in is when the moving platform of the manipulator precisely tracks a desired trajectory with respect to definite period, without need to interactive forces to be applied to the environment. A typical application for this situation can be realized in the motion of a flight simulator. In this case, it is required to define the desired path of the robot moving platform via analysis of many conditions and pilot commands, and then the manipulator has to determine the time history of actuator inputs needed to cause such motions [4][5].In what follows, a number of cases in motion control of the proposed model are analyzed and presented.

2. Kinematic Analysis of 4 RPR Manipulator

Referring to Figure 1 the *4 RPR* parallel robot is described as follows: In the base we have four points B_1, B_2, B_3 and B_4. In the moving platform, we have the same situation, four points P_1, P_2, P_3 and P_4.

Now each pair of points B_i and P_i is interconnected by a limb of length L_i which is an independent serial leg of type *RPR*, where *R* and *P* stands for revolute and prismatic joint, respectively. α_i stands for the limbs angles and Φ for the moving platform orientation. The motion of the platform is controlled by extending or retracting the actuated prismatic joints. A fixed frame O_B: X_BY_B is attached to the fixed base at point O_B, and the position of the center of the moving platform *o* is marked by $o=[x,y]$. Hence, the robot is a *3 dof* manipulator with a degree of redundancy in the actuators.

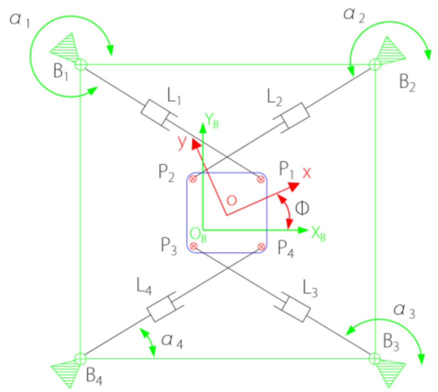

Figure 1. 4-RPR Kinematic Diagram.

3. Dynamic Analysis of 4 RPR robot: Newton Euler Formulation

This section is devoted to an overview on the dynamics of the 4 RPR parallel robot. Dynamic formulation of the limbs and the moving platform are carried out.

3.1 Dynamic formulation of the limbs

Figure 2 illustrate the free body representations of the limbs and the moving platform of the 4 RPR manipulator, where the forces at point B_i are $F_{Bi/S}$ and $F_{Bi/N}$, in which $\hat{N}$ is perpendicular to the limb and $\hat{S}$ is along its direction. In the same way, the forces at point P_i are represented by $F_{Pi/S}$ and $F_{Pi/N}$. and a_{mi} are the velocity and acceleration of the limb center of mass. We assume that $F_e = [f_{ex}, f_{ey}, r_e]^T$ is the only external disturbance force and moment acting on the robot. The moment of inertia in z-axis about the center of mass is denoted by I_{mi} and the limbs are considered rigid bodies with a mass *m*. The Newton Euler equations are given by:

$$\sum F_{ext} = m_i a_{mi} \quad (1)$$

$$\sum {}^{mi}\eta_{ext} = I_{mi}\ddot{\alpha}_i \quad (2)$$

Where $\sum F_{ext}$ is the sum of all external forces acting on each segment of the robot and $\sum {}^{mi}\eta_{ext}$ is the sum of external moments in the z direction about the center of mass of each segment *mi*.

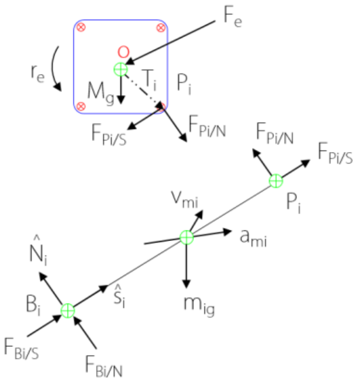

Figure 2. Free body representation.

$$F_{Pi/S} + F_{Bi/S} = \frac{\rho}{2}\left(2L_i\ddot{L}_i - (L_i\dot{\alpha}_i)^2 - 2L_i g_S\right) \quad (3)$$

$$F_{Pi/N} + F_{Bi/N} = \frac{\rho}{2}\left(L^2{}_i\ddot{\alpha}_i + 3L_i\dot{L}_i\dot{\alpha}_i - 2L_i g_N\right) \quad (4)$$

$$F_{Pi/N} - F_{Bi/N} = \frac{\rho}{6}L^2{}_i\ddot{\alpha}_i \quad (5)$$

Where $g = g_S\hat{S}_i + g_N\hat{N}_i + g_Z\hat{Z}$ is the gravitational acceleration, and its components can be derived from the following equations:

$$g_S = g.\hat{S}_i \ ; \ g_N = g.\hat{N}_i \ ; \ g_Z = g.\hat{Z} \quad (6)$$

From (3) through (5), $F_{Bi/N}$, $F_{Pi/N}$, $F_{Pi/S}$ and $F_{Bi/S}$ are determined as follows :

$$F_{Bi/N} = \rho\left(\frac{1}{6}L^2{}_i\ddot{\alpha}_i + \frac{3}{4}L_i\dot{L}_i\dot{\alpha}_i - \frac{1}{2}L_i g_N\right) (7)$$

$$F_{Pi/N} = \rho\left(\frac{1}{3}L^2{}_i\ddot{\alpha}_i + \frac{3}{4}L_i\dot{L}_i\dot{\alpha}_i - \frac{1}{2}L_i g_N\right) (8)$$

$$F_{Pi/S} = r_i + \rho\left(L_i\ddot{L}_i - \frac{1}{2}(L_i\dot{\alpha}_i) - L_i g_S\right) (9)$$

$$F_{Bi/S} = -r_i \ (10)$$

Where $F_{Bi/N}$ are the pivot reaction force, $F_{Pi/N}$ are the internal force between the segments and the platform, and $F_{Bi/S}$ are the actuator forces acting on the segments that can be considered as a tension force in the actuator denoted $r = [r_1, r_2, ..., r_i]^T$.

3.2 Dynamic formulation of the moving platform

The dynamic analysis of the platform is accomplished using the same free body representation in Figure 2. The Newton Euler equations of the platform are given by:

$$\sum F_{ext} = Mg + F_e - \sum_{i=4}^{4}\left(F_{Pi/S}\hat{S}_i + F_{Pi/N}\hat{N}_i\right) = Ma_O \quad (11)$$

$$\sum {}^{O}\eta_{ext} = r_e\hat{Z} - \sum_{i=1}^{4}T_i \times \left(F_{Pi/S}\hat{S}_i + F_{Pi/N}\hat{N}_i\right) = I_M\ddot{\Phi}\hat{Z} \quad (12)$$

Now equation (11) can be written as:

$$M\ddot{x}_O - M(g.\hat{x}) - F_{ex} + \sum_{i=4}^{4}\left(F_{Pi/S}\hat{S}_{ix} + F_{Pi/N}\hat{N}_{ix}\right) = 0 \quad (13)$$

The unit vector $\hat{S}_i$ is determined as follows $\hat{S}_i = \left[S_{ix}, S_{iy}\right]^T$, hence, $\hat{N}_i = \left[N_{ix}, N_{iy}\right]^T = \left[-S_{iy}, S_{ix}\right]^T$.

Finally:

$$M\ddot{x}_O - M(g.\hat{x}) - F_{ex} + \sum_{i=4}^{4}\left(F_{Pi/S}S_{ix} - F_{Pi/N}S_{iy}\right) = 0 \,(14)$$

$$M\ddot{y}_O - M(g.\hat{y}) - F_{ey} + \sum_{i=4}^{4}\left(F_{Pi/S}S_{iy} + F_{Pi/N}S_{ix}\right) = 0 \,(15)$$

$$I_M\ddot{\Phi} - r_e - \sum_{i=4}^{4}\left(F_{Pi/S}(T_{ix}S_{iy} - T_{iy}S_{ix}) + -F_{Pi/N}(T_i\hat{S}_i)\right) = 0 \quad (16)$$

These equations are leading the motion of the manipulator, where $F_e = \left[f_{ex}, f_{ey}, r_e\right]^T$ is the wrench applied on the platform, $F_{Pi/S}$ and $F_{Pi/N}$ the forces interacting between the segments and the platform are derived from the limbs dynamics in the previous section.

4. Motion Control of the 4 RPR Robot

In motion control of a parallel robot, we assume that the controller calculates the necessary actuator forces to make the robot motion pursue a desired trajectory.

We consider Equation (17) representing the general closed form dynamic model for a parallel manipulator:

$$M(X)\ddot{X} + C(X,\dot{X})\dot{X} + G(X) = F + F_e \quad (17)$$

Where $M(X)$, $C(X,\dot{X})$, $G(X)$ and $F + F_d$ are: the mass matrix, the Coriolis centrifugal matrix, the gravity vector and the generalized forces applied to the platform center of mass plus the disturbance wrench, respectively.

4.1 Controller Description

The control method introduced in this paper is a PD controller shown in Figure 3. This controller consists of *n* PD controllers performing on each error component, where n is the number of the actuators. The PD controller is represented by $K_D + K_P$ block. $F = [F_x \, F_y \, r_\Phi]^T$ stand for the output of the PD controller.

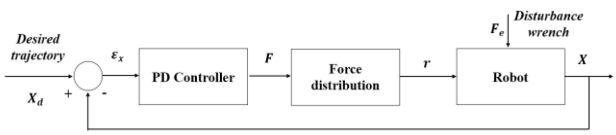

Figure 3. Block diagram containing robot and controller.

Using this configuration, each tracking error element denoted by $\varepsilon_x = [\varepsilon_x \, \varepsilon_y \, \varepsilon_\Phi]$, is treated individually by its

PD controller. The control effort is computed by the follow equation:

$F = K_d \dot{\varepsilon}_x + K_p \varepsilon_x$ (18)

The actuator forces can be generated by:

$r = J^{-T} F$ (19)

The implementation of the PD controller can be directly done in the joint space by equation:

$r = K_d \dot{\varepsilon}_q + K_p \varepsilon_q$ (20)

5. Simulations and results

The desired trajectory of the moving platform is generated and given in Figure 4.

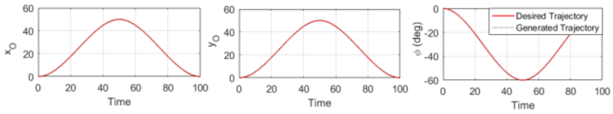

Figure 4: Desired and generated path of the manipulator for PD control.

Case 1: Without measurement noise

As seen in Figure 4, the desired trajectory and the final one generated by the closed loop are identical.

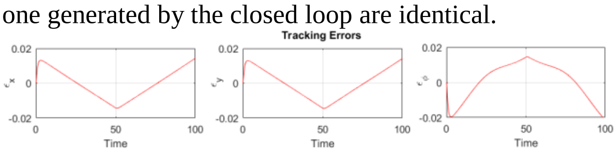

Figure 5: Tracking errors of the robot using PD controller.

Figure 5 shows that the PD controller is of minimizing the tracking errors to less than 0.013 in position and less than 0.019 in orientation.

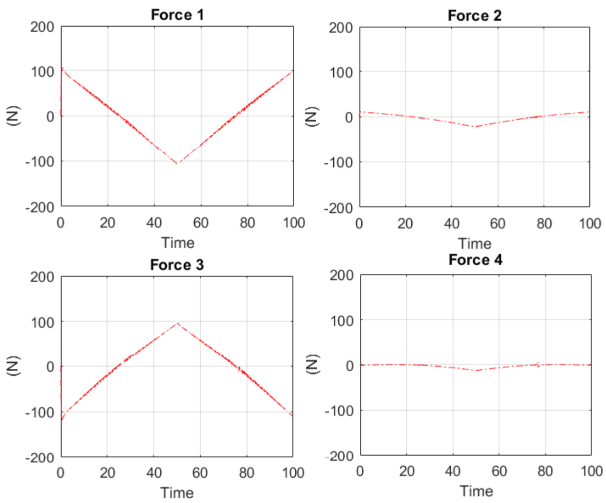

Figure 6: Actuator forces of the robot generated by the PD controller.

The actuator forces are illustrated in Figure 6. We can notice that since the robot moves in positive x and y directions the forces of the first and the third segments are dominant.

Case 2: With measurement noise

In this case, we consider that all the X variables are contaminated with a Gaussian noise. As we are using high gain PD controller, the quantity of noise is limited in this case. As presented in Figure 7 the tracking errors are not much increased compared to the case without noise.

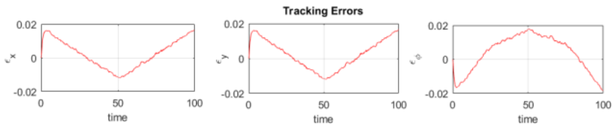

Figure 7: Tracking errors of the robot using PD controller with noise.

Figure 8 shows that the necessary actuator forces to perform such operation are very oscillatory. Considering that high gains provided by PD controllers are used to adjust the tracking errors. The amplitude of noise is 0.01% of the original signal, also the PD controller gain is higher than 10^4. So practically producing such actuator force is not feasible; hence, the tracking errors shown in Figure 7 are not possible.

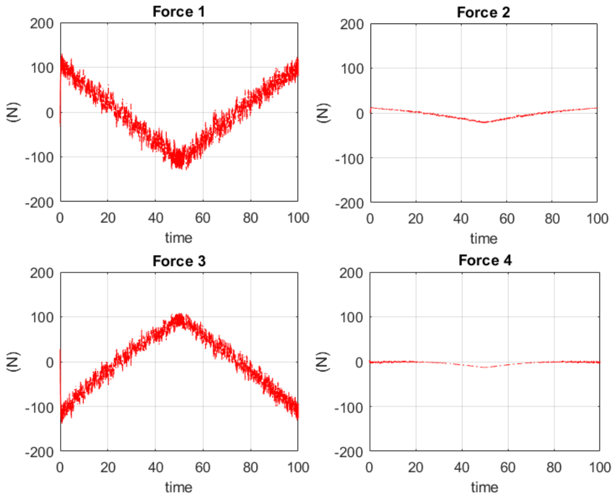

Figure 8: Actuator forces of the robot generated by the PD controller in presence of noise.

6. Conclusion

In this paper a dynamic study for the 4 RPR parallel robot is presented. A proportional–derivative controller architecture is described and applied on the dynamic model in order to analyze the motion control performances of the proposed structure.

References

[1] E. Abedloo, A. Molaei, H. D. Taghirad, and S. Member, “Closed-Form Dynamic Formulation of Spherical Parallel Manipulators by Gibbs-Appell Method,” pp. 576–581, 2014.

[2] H. Høifødt, “Dynamic Modeling and Simulation of Robot Manipulators,” no. June, p. 112, 2011.

[3] W. Ibrahim, O., Khalil, “Inverse and direct dynamic models of Hybride robots.,” *Mech. Mach. Theory* , vol. 45(4), pp. 627–640, 2010.

[4] F. Wobbe, M. Kolbus, and W. Schumacher, “Enhanced Motion Control Concepts on Parallel Robots.”

[5] L. Vermeiren, A. Dequidt, M. Afroun, and T. Guerra, “Motion control of planar parallel robot using the fuzzy descriptor system approach,” *ISA Trans.*, vol. 51, no. 5, pp. 596–608, 2012.