Nonlocal meshless continuum model for vibration of double layer graphene sheets rest on elastic medium

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Abstract

The present work investigates the nanoscale vibration behavior of double-layered graphene sheets (DLGSs) resting on elastic medium taking into account effects of van der Waals (vdW) interactions. The analysis is carried out employing meshless method (MLSA) based on the nonlocal elastic plate theory. The DLGSs is modeled by a double thin elastic nanoplates continuum model which account for the small scale effect. The collocation method is adopted to solve the strong form of equilibrium equations and enforcing the boundary conditions. The efficiency of the proposed approach is demonstrated by a comparison with an exact solution computed by the Navier's method. The effects of aspect ratio, stiffness of surrounding elastic medium and nonlocal parameter on the vibration behavior of (DLGSs) are performed.

Keywords: *Vibration, double-layered graphene sheets, nonlocal elastic plate theory, van der Waals interaction, elastic medium, Winkler-Pasternak foundation, Meshless method, (MLSA), collocation method.*

1. Introduction

Recently, nanomaterials such as Graphene sheets (GSs) have attracted considerable attention of the scientific community due to their superior properties. There exist three main categories for the modelings and simulations of nanomaterials: atomistic and molecular modeling, continuum mechanics and a combination of atomistic modeling and continuum mechanics. One of the updated continuum mechanics methods for analysis of nanostructures is the nonlocal elasticity theory proposed by Eringen [1]. In some cases, the analytical solution can be obtained when applying the nonlocal continuum mechanics, but in others, in which certain physical complexities cannot be interpreted or described by way of simple analytical expression, a numerical technique is needed. Certainly, the numerical simulation of nanostructures is usually made, in computational mechanics, using the mesh-based methods. Generating meshes with internal complex structures and external boundaries for the purpose of solving nanostructures problem using mesh-based methods is a difficult task. As alternative approaches to mesh-based method, new methods, qualified" mesh-free or meshless methods" have developed in recent years [2]. These methods do not require a mesh to build the approximation of the unknown field in the area but only a point cloud.

In this work we develop a novel efficient nonlocal approach to study the free vibration of double-layered graphene sheets (DLGSs) taking into account the effects of small length scales and the van der Walls interactions between the two layers. It is built by coupling a meshless method based on the Moving Least Squares Approximation (MLSA) and an elastic nanoplate nonlocal theory.

2. Problem statement

Let us consider a double layered graphene (DLGSs) in which each GSs has length L_1, width L_2, thickness h, mass density ρ, Young's moduli E_1, E_2 and Poisson's ratio ν_{12}, ν_{21}. The two layers of DLGSs interact via van der Waals forces and are surrounded by an external elastic medium as shown in (fig.1 (a)) and subjected to transverse load q. A Cartesian coordinate system (O, x_1, x_2, x_3) is used for (GSs).

2.1 Mechanical modeling

The double layered graphene sheets (DLGSs) are modeled as a thin double orthotropic elastic nanoplates. The elastic medium is modeled by Winkler-Pasternak foundations represented by continuously distributed vertical springs with stiffness coefficients k_w and k_p depicted in fig.1 (b).

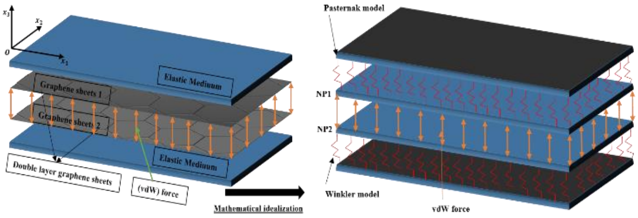

Figure. 1- (a) Double-layered graphene sheets (DLGSs) surrounded by on an elastic medium, (b) Double elastic nanoplates resting in elastic Winkler-Pasternak foundations

2.2 Nonlocal constitutive relations of elastic continuum orthotropic nanoplate

According to the nonlocal elasticity theory proposed by Eringen [1], the stress at a reference point x of a body is a function of the strain field at every point x' in the medium. The stress tensor of the nonlocal elasticity is defined by

$$\mathcal{L}\sigma_{ij}^{nl} = \sigma_{ij}^{l} \qquad (1)$$

where $\mathcal{L} = (1 - \mu\nabla^2)$ is a the linear differential operator and $\mu = (e_0 a)^2$ is the nonlocal parameter, ∇^2 is the Laplacian operator, a is an internal characteristic length, e_0 is a nonlocal scaling parameter, σ_{ij}^{l} is the local stress tensor related to the strain tensor by the generalized Hooke's law.

2.3 Van der Waals force between layers and applied external load

The resultant effective transverse load applied on the jth layers can be written as:

$$q_e^{(j)} = q - c\left(u_3^{(j)} - u_3^{(i)}\right) - k_w u_3^{(j)} + k_{p\gamma} u_{3,\gamma\gamma}^{(j)} \quad (2)$$

where k_w, $k_{p\gamma}$ and c are respectively the Winkler spring, Pasternak shear in two orthogonal directions, $\boldsymbol{c}$ the van der Waals coefficient, and u_3 is the transverse displacement, $\gamma\gamma$ designates the two directions $(\gamma = 1,2)$.

2.4 Governing dynamic equations

The nonlocal governing equations of motion for each orthotropic nanoplate $(j = 1,2)$ coupled to an elastic medium are expressed by:

$$-D_{\alpha\beta\gamma\delta} u_{3,\gamma\delta\alpha\beta}^{(j)} + \mathcal{L}\left[q_e^{(j)} - I_0 \ddot{u}_3^{(j)} + I_2 \ddot{u}_{3,\alpha\alpha}^{(j)}\right] = 0 \quad (3)$$

Where the inertia terms is defined as $I_k = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho z^k dz$, $(k = 0,2)$ and $D_{\alpha\beta\gamma\delta}$ are the bending rigidities given by $D_{\alpha\beta\gamma\delta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \hat{c}_{\alpha\beta\gamma\delta} z^2 dz$, $q_e^{(j)}$ represents the distributed load and $\hat{c}_{\alpha\beta\gamma\delta}$ are the elastic modulus components.

2.5 Boundary condition

All edges of nanoplates are simply supported (SSSS):

$$\mathrm{u}_3^{(j)} = \mathrm{u}_{3,ii}^{(j)} = 0, in\ x_i = 0, x_i = L_i \quad j = 1,2, i = 1,2 \quad (4)$$

2.6 Exact analytical expressions of vibration natural frequencies [4]

The developed coupled equations eq. (3) have been solved by Navier's approach for (SSSS) boundary conditions. The frequencies can be calculated as:

$$\omega_{mn}^1 = \sqrt{\frac{D_e}{\Gamma_{mn}\Lambda_{mn}}} \text{ and } \omega_{mn}^2 = \sqrt{\frac{D_e + 2c\Lambda_{mn}}{\Gamma_{mn}\Lambda_{mn}}} \quad (5)$$

with $\Gamma_{mn} = I_0 + I_2(\alpha^2 + \beta^2)$, $\Lambda_{mn} = 1 + \mu(\alpha^2 + \beta^2)$ and $D_e = \left(D_{11}\alpha^4 + 2\bar{D}_{12}\alpha^2\beta^2 + D_{22}\beta^4 + k_w\Lambda_{mn} + k_p\Lambda_{mn}(\alpha^2 + \beta^2)\right)$ with $\alpha = \left(\frac{m\pi}{L_1}\right), \beta = \left(\frac{n\pi}{L_2}\right)$, $D_{ij}(i,j = 1,2)$ are the bending stiffnesses of orthotropic double layer graphene sheets.

3. Solution procedure

3.1 MLSA Discretization.

The displacements field of the (DLGSs) can be expressed according to the (MLS), as follows:

$$\left\{u_3^{(j)}\right\} = \sum_{I=1}^{n} \boldsymbol{\Phi}_I(x_\delta)\left\{u_{3I}^{(j)}\right\} \ (j = 1,2), (\delta = 1,2), \quad (6)$$

where n is the total number of nodes used to discretize the domain, $\boldsymbol{\Phi}_I$ and u_{3I} denote the shape function of the (MLSA) and displacement value for each nanoplates associated with node I, respectively, assuming the nodal parameter has the following motion expression $\langle u_{zI}^1, u_{zI}^2\rangle^T = \{\bar{\mathrm{X}}\}\boldsymbol{e}^{\boldsymbol{iwt}} = \langle \bar{u}_z^1, \bar{u}_z^2\rangle^T \boldsymbol{e}^{\boldsymbol{iwt}}$, where i is the imaginary unit, t indicates time, $\langle \bar{u}_z^1, \bar{u}_z^2\rangle^T$ is the eigenvector and ω is natural frequency, after same manipulation we obtain the fallowing equation:

$$(-\omega^2[M] + [K])\{\bar{\mathrm{X}}\} = 0. \quad (7)$$

in which the stiffness and mass matrix are defined by $[K] = [C]_I \cdot [B]_I$, $[M] = [C]_{II} \cdot [B]_{II}$, where $[C]_I$, and $[C]_{II}$ are matrices dependent on coefficients of the elasticity stress tensor and (vdW) force coefficients, $[B]_I$ and $[B]_{II}$ are the matrices of derivatives of MLS shape functions.

4. Validation of the proposed approach

4.1 Vibration of a square double layered graphene (DLGSs)

Consider a square double layered graphene (DLGSs) embedded in an elastic homogeneous medium of side $L = L_1 = L_2 = 10.2nm$ and thickness $h = 0.34nm$, having the material properties [2]: Young's modulus $E_1 = 1765GPa$, $E_2 = 1588GPa$, Poisson's ratio $\nu_{12} = 0.3$, $\nu_{21} = 0.27$, mass density $\rho = 2.3g/cm^3$. The considered numerical values of Winkler spring k_w, Pasternak shear k_p and the van der Waals coefficient c are respectively: $k_w = 1.13\ 10^{18}Pa/m, k_p = 1.13Pa/m, c = 4.15\ 10^{19}Pa/m$. As the elastic medium is homogeneous then $k_{p1} = k_{p2}$. The nonlocal parameter μ belonging to the range $[0, 2nm^2]$.The (DLGSs) is assumed to be free from any in-plane or transverse loadings $q = 0$. The considered number of grid points is $11x11$.

4.2 Numerical results and discussions

4.2.1 Natural frequencies of (DLGSs)

The numerical results for natural frequencies of (DLGSs) computed by the proposed approach (MLS) solution compared to the exact solution (eq. (5)) are presented in Table. 1.

$\boldsymbol{\mu}$	Natural frequencies	Exact Solution(GHz)	MLS Solution(GHz)
0	$\boldsymbol{\omega^1}$	0.03839	0.03876
	$\boldsymbol{\omega^2}$	0.33132	0.33659
1	$\boldsymbol{\omega^1}$	0.04568	0.04504
	$\boldsymbol{\omega^2}$	0.39418	0.38254
2	$\boldsymbol{\omega^1}$	0.05296	0.05225
	$\boldsymbol{\omega^2}$	0.45704	0.44326

Table. 1 - Comparison of natural frequencies (GHz) with the exact solution [3] for a (SSSS) square (DLGSs).

The obtained numerical results are in good agreement with the analytical solution given by (eq. (5)).

4.2.2 Effect of nonlocal parameter and length on natural frequencies of (DLGSs)

In order to illustrate the small scale and length effects on natural frequencies of (DLGSs), we define frequency ratio as: $Frequency\ ratio = \frac{\omega_{nl}}{\omega_l}$. The variation of frequency ratio with length is plotted in Figure. 2, for simply supported square orthotropic double layered graphene sheets, having the following proprieties [4], $E_1 = E_2 = 1.06\,TPa$, $\nu_{12} = \nu_{21} = 0.25$, $\rho = 2250 kg/m^3$, $c = 4.1510^{19}\,Pa/m$ for each nanoplate. The results, in the absence of elastic foundation, have been shown for different values of nonlocal parameters ($0\,nm^2 - 3\,nm^2$) and different values of length ($5\,nm - 30 nm$). This plot shows that lower ratio is obtained at higher value of nonlocal parameter for the 1st (top nanoplate) and 2nd (bottom nanoplate), and the frequency ratio increase with length.

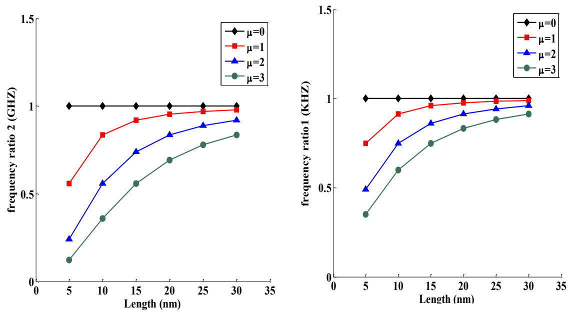

Figure. 2 - Variation of frequency ratio for the 1st and 2nd nanoplate with length.

4.2.3 Effect of elastic foundation on natural frequencies of (DLGSs)

In this subsection, we have investigated the influence of surrounding medium on the natural frequencies of (DLGSs). The considered numerical data are [5]: Young's moduli $E_1 = 1465\,GPa$, $E_2 = 1588\,GPa$, Poisson's ratio $\nu_{12} = \nu_{21} = 0.30$, mass density $\rho = 2250 kg/m^3$, van der Waals coefficient $c = 4.15\ 10^{19}\ Pa/m$. First, we have considered ($k_w = 200$) with simply supported edge condition, next, we have taken ($k_p = 0$) with the same edge condition.

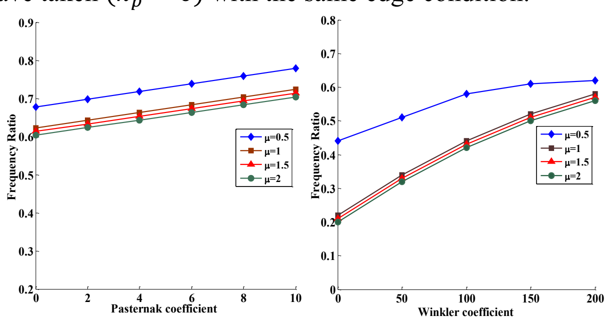

Figure .3 -Effect of Winkler-Pasternak coefficient on frequency ratio.

Different values of nonlocal parameters are considered. It is observed from the figure.3, that frequency ratio increases linearly by increasing the stiffness of the elastic foundation either through the springy (Winkler coefficient) or the shear effect (Pasternak coefficient), we can also conclude that the stiffness of surrounding elastic medium increases when the nonlocal effect decreases.

5. Conclusion

The free vibration of double-layered graphene sheets (DLGSs) surrounded by an elastic medium has been analyzed by an approach which combines a nonlocal continuum model of classical plate theory and a meshless method based on MLS approximation (MLSA). The considered (MLSA) is based on a strong formulation of problem in order to avoid numerical integrations. The collocation method is adopted for the derivation of discrete system and enforcing boundary conditions. A comparison to an analytical solution shows the effectiveness of the proposed approach. The effects of length, nonlocal parameter and elastic foundation on the vibration behavior of (DLGSs) are examined. It is observed that the nonlocal effect depends on surrounding elastic medium. As the stiffness of surrounding elastic medium increases, the non local effect decreases and increases as the size of the graphene sheet is decreased.

6. References

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