

# Effects of generative / destructive chemical reaction and temperature dependent viscosity of non-Newtonian fluids on dual-diffusion convection in a porous medium

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## Abstract :

In this work we study the combined free convection, due to thermal and species diffusion, of a viscous incompressible non Newtonian fluid over a vertical plate embedded in a saturated porous medium with three thermal states of the surface and a constant concentration in the presence of a chemical reaction. The effect of temperature dependent viscosity is also investigated. The Ostwald-de Waele power-law model is used to characterize the non-Newtonian fluid behavior. The governing boundary layer equations along with the boundary conditions are first cast into a dimensionless form by a unique similarity transformation and the resulting coupled differential equations are then solved numerically by a computational program based on the fifth order Runge-Kutta scheme with shooting iteration technique. The results are illustrated and the physical aspect is discussed for temperature and concentration profiles, as well as the Nusselt and Sherwood numbers for various values of the parameters, which govern the problem.

**Key words :** *Dual-diffusion convection, non-Newtonian fluid, similarity method, porous medium*

## 1 Introduction

The heat and mass transfer for non-Newtonian fluids in porous media, in general, is of great pragmatic importance in a wide variety of scientific and engineering applications such as geothermal fields, fibrous insulation, oil recovery, food processing, spreading of chemical contaminants through water-saturated soil and many others. The growing need for chemical reactions in chemical engineering and hydrometallurgical industries requires the study of heat and mass transfer with chemical reaction. A comprehensive review of the literature concerning double-diffusive natural convection in a fluid saturated porous media can be found in the review articles of

Trevisan and Bejan [1], Merkin [2], Hady et al. [3] and in the recent book of Nield and Bejan [4]. The majority of the previous studies are based on the constant physical parameters of the fluid. For most realistic fluids, the viscosity shows a rather pronounced variation with temperature. Thus it is necessary to take into account the variation of viscosity with temperature in order to accurately predict the heat and mass transfer rates. This study extends the work of Moorthy and Senthilvadivu [5] for dual-diffusion free convection along a vertical flat plate embedded in Darcy porous medium saturated with non-Newtonian fluids for three thermal states of the surface and a constant concentration.

## 2 Physical and mathematical models

Our study is to investigate the double diffusive free convection induced by a heated vertical impermeable plate embedded in a saturated porous medium. The physical model is sketched in figure 1.

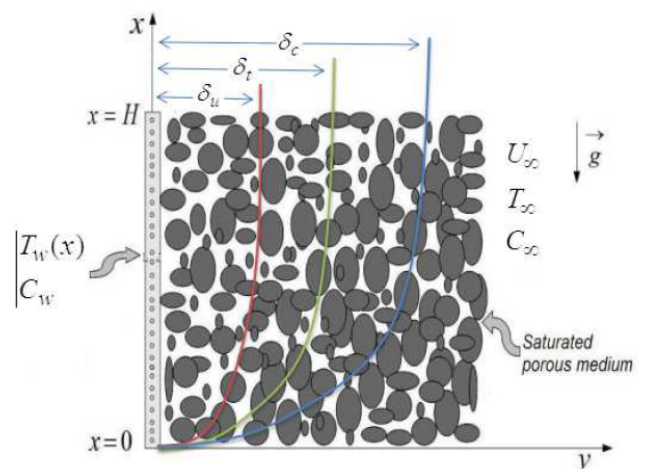


FIGURE 1 – Physical configuration and coordinate system.

The governing equations describing the fluid flow can be written as follows :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$|u|^{n-1} u = \frac{\rho g \beta_t K(n)}{\mu} (T - T_\infty) + \frac{\rho g \beta_c K(n)}{\mu} (C - C_\infty), \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}, \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 (C - C_\infty)^m, \quad (4)$$

The appropriate boundary conditions of our problem are

$$v = 0, \quad T = T_\infty + A x^\lambda \quad C = C_w \quad \text{at } y = 0, \quad (5)$$

and

$$u = U_\infty = 0, \quad T = T_\infty \quad C = C_\infty \quad \text{as } y \rightarrow \infty, \quad (6)$$

We introduce the following dimensionless similarity variable  $\eta$  and the similarity functions  $f(\eta), \theta(\eta)$  and  $\phi(\eta)$  as

$$\left\{ \begin{array}{l} \eta = \frac{y}{x} Ra_x^{1/2}, \\ \psi = \alpha Ra_x^{1/2} f(\eta), \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \\ Ra_x = \left[ \frac{\rho g \beta_t K(n) (T_w - T_\infty) x^n}{\mu_\infty \alpha^n} \right]^{1/n} \end{array} \right. \quad (7)$$

where  $\psi$  is the stream function defined by  $u = \partial\psi/\partial y$  and  $v = -\partial\psi/\partial x$ , which identically satisfies the continuity Eq. (1), and  $Ra_x$  is the local Rayleigh number for the power-law fluids. In terms of these new variables, the momentum, energy and mass diffusion equations together with the boundary conditions become,

$$(f')^n + (\theta + N\phi) \left( \frac{\theta - \theta_r}{\theta_r} \right) = 0 \quad (8)$$

$$\theta'' + \frac{1}{2} \left( \frac{\lambda + n}{n} \right) f \theta' - \lambda \theta f' = 0 \quad (9)$$

$$\frac{1}{Le} \phi'' + \frac{1}{2} \left( \frac{\lambda + n}{n} \right) f \phi' - \delta \phi^m = 0 \quad (10)$$

and

$$f = 0, \quad \theta = 1 \quad \phi = 1 \quad \text{at } \eta = 0, \quad (11)$$

$$f' = 0, \quad \theta = 0 \quad \phi = 0 \quad \text{as } \eta \rightarrow \infty,$$

where the primes in Eqs.(8)-(10) denote differentiation with respect to the similarity variable  $\eta$ . Here the parameters  $N$ ,  $Le$  and  $\delta$  are the sustentation parameter, the Lewis parameter and the reaction rate parameter respectively. They are defined by

$$\begin{aligned} N &= \frac{\beta_c (C_w - C_\infty)}{\beta_t (T_w - T_\infty)}, \\ Le &= \frac{\alpha}{D}, \\ \delta &= \frac{k_1 (C_w - C_\infty)^{m-1} x^2}{\alpha Ra_x} \end{aligned} \quad (12)$$

Also,  $\theta_r$  is the viscosity parameter defined by

$$\theta_r = \frac{T_e - T_\infty}{T_w - T_\infty} = -\frac{1}{r (T_w - T_\infty)}, \quad (13)$$

It is worth mentioning here that for  $r \rightarrow 0$  i.e.  $\mu = \mu_\infty$  (constant) then  $\theta_r \rightarrow \infty$ . It is also important to note that  $\theta_r$  is negative for liquids and positive for gases.

The parameter  $\delta$  in the species diffusion Eq.(10) represents the following three situations :  $\delta > 0$ , for destructive chemical reaction ;  $\delta < 0$ , for generative chemical reaction ; and  $\delta = 0$ , for no reaction. This study is focused on three different cases : the first case ( $\lambda = 0$ ) corresponds to isothermal surface ; the second case ( $\lambda = 1$ ) represents linear temperature along the vertical surface ; and the third case ( $\lambda = 1/2$ ) represents square root temperature variation along the surface.

In equation (2) the permeability of the porous medium  $K$  for flows of non-Newtonian power-law fluids is given by the relationships provided by Christopher and Middleman [6].

The parameters of the engineering interest for the present problem are the local Nusselt number and the Sherwood number, which are given by the following expressions :

$$\begin{aligned} \frac{Nu_x}{Ra_x^{1/2}} &= -\theta'(0), \\ \frac{Sh_x}{Ra_x^{1/2}} &= -\phi'(0) \end{aligned} \quad (14)$$

### 3 Solution method and results analysis

The set of coupled nonlinear ordinary differential Eqs. (8)-(10), subject to the boundary conditions (11), are solved numerically for three thermal states of the surface mentioned earlier by using the fifth order Runge-Kutta scheme accompanied by the soothing method. A uniform step size of  $\Delta\eta = 0.001$  is found to be satisfactory to give results that converge to within an error of  $10^{-6}$  in nearly all the cases. The value of  $\eta_\infty$  was chosen as large as possible, without causing numerical instability in the resulted  $f'$ ,  $\theta$  and  $\phi$ .

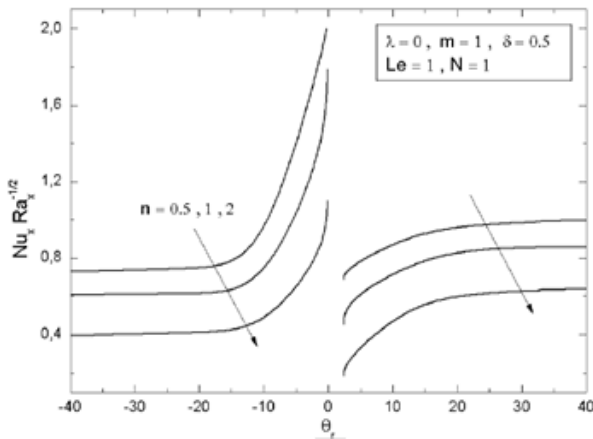


FIGURE 2 –  $Nu_x$  distributions versus  $\theta_r$  for various values of  $n$ .

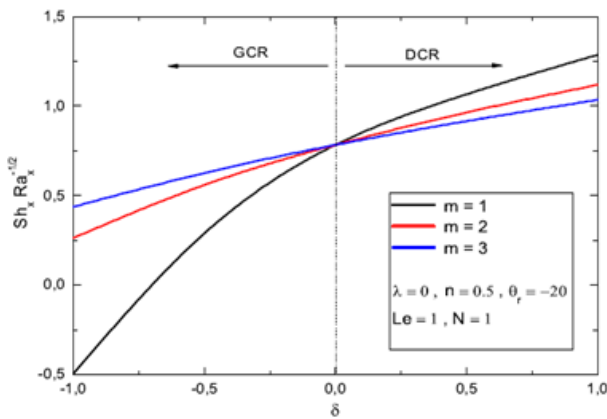


FIGURE 3 –  $Sh_x$  distributions versus  $\delta$  for various values of  $m$ .

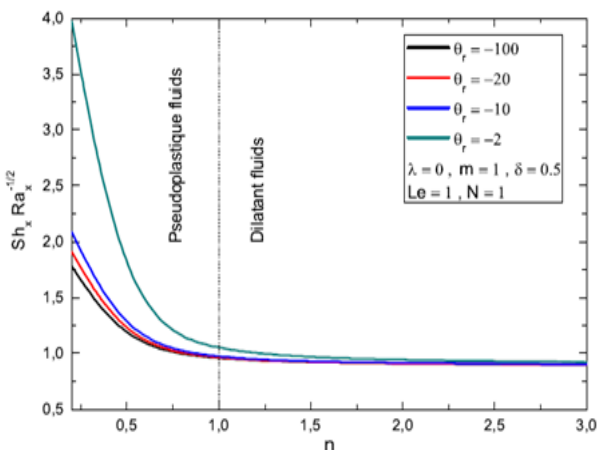


FIGURE 4 –  $Sh_x$  distributions versus  $n$  for various values of  $\theta_r < 0$ .

From figure 2, it is clearly remarkable that the rate of heat transfer at the surface is more important in the case of pseudoplastic fluid than in the case of dilatant fluid

regardless of fluid viscosity. Furthermore, it is obvious that in the case of liquids ( $\theta_r < 0$ ) the heat transfer rate is more than that for the constant viscosity case. The opposite behavior has been observed in the case of gases ( $\theta_r > 0$ ).

Figure 3 shows the species transfer rate at the wall, for an isothermal and impermeable plate buried into a pseudo-plastic fluid saturated porous medium ( $n = 0.5$ ,  $\theta_r = -20$ ), according to the chemical reaction parameter for three values of the chemical reaction order ( $m = 1, 2$  and  $3$ ). Here, it is observed that the species transfer rate increases with increasing chemical reaction parameter  $\delta$  for all values of the parameter  $m$  considered. Also, it is important to note that this transfer rate increases with increasing  $m$  in the presence of a generative chemical reaction and decreases in the case of a destructive chemical reaction.

Figure 4 describes the mass transfer rate at the surface of an isothermal and impermeable plate, embedded into a pseudoplastic fluid saturated porous medium ( $n = 0.5$ ), according to the parameter  $n$  for selected values of the viscosity parameter  $\theta_r (< 0)$  in the presence of the first order destructive chemical reaction ( $\delta = 0.5$ ). One can easily conclude that the mass transfer rate at the wall is very important when  $\theta_r \rightarrow 0 (< 0)$  for pseudoplastic fluids ( $n < 1$ ) but no significant influence of the viscosity parameter on the mass transfer rate is shown for dilatant fluids. For all values of  $\theta_r$  considered, the local Sherwood number near the leading edge decays sharply as the value of the parameter  $n$  moves from the range of the pseudoplastic fluids to the range of the dilatant fluids.

## 4 Conclusions

From the present numerical investigation we conclude that :

- The effect of a variable viscosity on the temperature distribution is significant for pseudoplastic fluids than for dilatant fluids.
- The generative first order chemical reaction effect ( $\delta < 0$ ) causes thicker concentration boundary layer while the destructive first order chemical reaction effect ( $\delta > 0$ ) has the tendency to reduce the thickness of concentration boundary layer.
- The heat transfer rate is more in the case of liquids when the variable viscosity is considered as compared to the constant viscosity case. In the case of gases, it is less than the constant viscosity case.
- The rate of heat and mass transfer are more important in the case of pseudoplastic fluids than in the case of dilatant fluids as  $\theta_r \rightarrow 0$  for liquids ( $\theta_r < 0$ ).

- The species transfer rate increases with increasing chemical reaction parameter  $\delta$ . Also, this rate is amplified with increasing  $m$  in the presence of a generative chemical reaction and reduced in the case of a destructive chemical reaction.

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