SORET EFFECT ON SUBCRITICAL NATURAL CONVECTION INDUCED WITHIN A HORIZONTAL SATURATED POROUS ENCLOSURE SUBJECT TO LATERAL HEAT AND MASS FLUXES

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Abstract

The aim of this study is to examine the impact of the thermo-diffusion phenomenon, known as the Soret effect, on double diffusive natural convection induced in a horizontal Darcy porous enclosure saturated with a binary mixture. The considered porous cavity is subject to uniform lateral heat and mass fluxes. The long horizontal walls of this medium are considered adiabatic and impermeable to mass transfer. The Soret effect on the fluid flow properties and heat and mass transfer characteristics is discussed here for a particular situation characterized by a specific relationship between the solutal to thermal buoyancy forces ratio and the Soret parameter for which the rest state is a solution of the problem.

Key words: *Soret effect, Natural convection, Horizontal cavity, Lateral heat and mass fluxes, Analytical and numerical study, Subcritical convection.*

1. Introduction

Recent interest in the study of Soret effect on double-diffusive natural convection developed in a binary mixture has been motivated by its diverse applications in several natural, environmental and industrial processes. This interest finds also its justification by the different behaviours (multiple solutions, hysteresis and Hopf's bifurcations) developed due to this phenomenon. In order to analyze these behaviours, experimental and theoretical efforts have been devoted in the past. The experimental aspect of the problem was addressed [1-3] and numerous teams involved in the experimental aspect of thermodiffusion are cited in the review papers [4-5], giving an idea on the different technics used to measure the thermodiffusion coefficient and the difficulties inherent in such manipulations.

From a theoretical point of view, Soret-driven thermosolutal convection induced in a shallow porous layer subject to a vertical temperature gradient was studied analytically and numerically by Bourich et al. [6]. Using a linear stability analysis, the onset of over stability was predicted in this study. A comparative investigation for Darcy porous and clear fluid media were performed by the authors. The onset of Soret-driven convection in an infinite porous layer saturated by a binary fluid with impermeable horizontal walls maintained at different and uniform temperatures was investigated by Sovran et al. [7]. The criteria for the onset of motion via a stationary and Hopf's bifurcations were determined using a linear stability analysis. They showed that the bifurcation from the rest state depend, among other factors, on the separation ratio. Soret driven thermosolutal convection in a shallow Brinkman porous layer, with a stress-free upper surface, subject to constant fluxes of heat on its horizontal walls was studied analytically and numerically by Er-Raki et al. [8]. They found that, depending on the sign of the separation parameter, the Soret effect can play a stabilizing or a destabilizing role. Supercritical and subcritical Rayleigh numbers were determined as a function of the parameters governing the problem. Thermodiffusion phenomenon induced in a horizontal porous enclosure was examined previously also, under various thermal and solutal boundary conditions, by Bahloul et al. [9] and Bourich et al. [10] and many others.

Most of the studies addressing double diffusive natural convection combined with Soret effect within horizontal porous enclosures, have considered the situation where all the boundaries are impermeable to mass transfer and the flows are driven by the buoyancy effects due to temperature variations alone. In the present study, we investigate thermo diffusion effect on thermo-solutal natural convection induced in a horizontal shallow porous enclosure subject to lateral heat and mass fluxes. An approximate analytical solution of the problem, validated numerically, is presented. The Soret effect on the fluid flow properties and heat and mass transfer characteristics is also discussed in this study for the specific case $N = 1/(S_P-1)$.

2. Physical problem

The configuration under study is a two-dimensional horizontal shallow porous layer of height H' and width L' filled with a binary mixture (Fig. 1). The short vertical walls of the layer are subject to uniform fluxes of heat, q', and mass, j', while its horizontal long walls are considered adiabatic and impermeable to mass transfer.

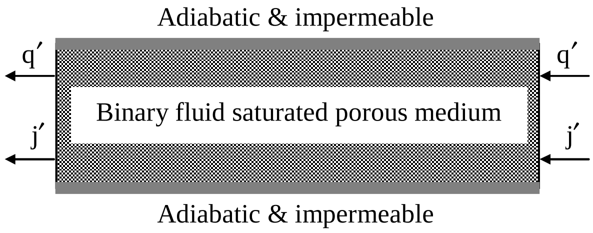

Figure 1: Schematic of the studied configuration.

The porous matrix is assumed isotropic and homogeneous and the Darcy law is adopted. The diluted binary solution that saturates the porous medium is modeled as a Boussinesq incompressible fluid.

Using the vorticity-stream function formulation, the dimensionless equations governing the Darcy model in the presence of Soret effect are as follows:

$$\eta \frac{\partial \zeta}{\partial t} + \zeta = R_T \left(\frac{\partial T}{\partial x} + N \frac{\partial S}{\partial x} \right) \quad (1)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \nabla^2 T \quad (2)$$

$$\varepsilon \frac{\partial S}{\partial t} + u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} = \frac{1}{Le} \left(\nabla^2 S + S_P \nabla^2 T \right) \quad (3)$$

$$\nabla^2 \psi = -\zeta \quad (4)$$

$$u = \frac{\partial \psi}{\partial y} \quad ; \quad v = -\frac{\partial \psi}{\partial x} \quad (5)$$

The associated boundary conditions are given by:

$$\left. \begin{array}{l} x = \pm A_r/2 \ : \ \psi = 0, \ \dfrac{\partial T}{\partial x} = 1, \ \dfrac{\partial S}{\partial x} = 1 - S_P \\ y = \pm 1/2 \ : \ \psi = 0, \ \dfrac{\partial T}{\partial y} = 0, \ \dfrac{\partial S}{\partial y} = 0 \end{array} \right\} \quad 6)$$

where u, v, ζ, ψ, T and S represent respectively the dimensionless horizontal and vertical components of the velocity, vorticity, stream function, temperature and concentration.

The governing parameters resulting from the adimensionalization of the above equations are the thermal Rayleigh number, $R_{T;}$ the Lewis number, Le; the Soret parameter, S_P; and the solutal to thermal buoyancy ratio, N. They describe respectively the thermal driving force, the relative importance of the thermal diffusivity with respect to the solute one, the thermo-diffusion phenomenon and the importance of solutal buoyancy forces due to the applied mass flux.

The Nusselt and Sherwood numbers are calculated using temperature and concentration differences between two contiguous vertical sections in the central part of the enclosure to avoid the effect of the edge effects:

$$Nu(y) = \lim_{\delta x \to 0} \delta x / \delta T(y) = 1/(\partial T / \partial x)_{x=0} \quad (7)$$

$$Sh(y) = \lim_{\delta x \to 0} \delta x / \delta S(y) = 1/(\partial S / \partial x)_{x=0} \quad (8)$$

where δx is the distance between these sections.

Then, the mean Nusselt and Sherwood numbers at different locations are calculated by the expressions:

$$\overline{Nu} = \int_{-1/2}^{1/2} Nu(y)\,dy \quad \text{and} \quad \overline{Sh} = \int_{-1/2}^{1/2} Sh(y)\,dy \quad (9)$$

These expressions of Nusselt and Sherwood numbers are used in the numerical calculations to avoid the effect of the end regions where the analytical solution is not valid.

3. Results and discussion

The mathematical results, corresponding to the specific case N = 1/(S_P-1), has shown that the present problem has one stable solution ($\propto R_T^{1/3}$) and an other instable ($\propto R_T^{-1}$) for which the corresponding flows rotate in the same direction. At large values of R_T (R_T>>1), these solutions are given by the following expressions:

$$\psi_0 \approx \sqrt[3]{\frac{120\left(Le^2 + (S_P Le + 1)/(S_P - 1)\right)}{A}} R_T^{1/3} \quad (10)$$

$$\psi_0 \approx \frac{D}{120\left(Le^2 + (S_P Le + 1)/(S_P - 1)\right)} R_T^{-1} \quad (11)$$

With $A = 512 Le^2$ and $D = 1800$

Similarly, it was subsequently demonstrated that no supercritical bifurcation is possible; only a subcritical bifurcation exists and the latter occurs at $R_T = R_{TC}^{sub}$ given by:

$$R_{TC}^{sub} = \frac{\tilde{\psi}_0 \left(4A\tilde{\psi}_0^2 + 2B\right)}{120\left|Le^2 + (S_P Le + 1)/(S_P - 1)\right|} \quad (12)$$

Where $\tilde{\psi}_0 = \sqrt{\left(\dfrac{-B + \sqrt{\Delta}}{6A}\right)}$

With $\Delta = B^2 + 12AD$ and $B = 960(Le^2 + 1)$

The Soret effect is illustrated by presenting in Figs. (2) and (3) the evolutions of Nu and Sh vs. S_P for Le = 3 and R_T = 100. The range [-4, 4], corresponding to the variation of S_P, is selected to include both critical values of S_P; 1-1/Le (0.666 for this case) and 1, in the vicinity of which the fluid flow and heat and mass transfers magnitudes undergo important changes. Also, in this range of S_P, a change in the flow direction occurs. At first sight, Figs. 2-3 show that the quantities Nu and Sh corresponding to the stable branches vary qualitatively in the same way. More precisely, they decrease with S_P for both $S_P \le S_P^{cr1}$ and $S_P > 1$ by exhibiting asymptotic evolutions at large $|S_P|$ and increase with this parameter in the range $S_P^{cr2} \le S_P < 1$. When S_P approaches 1 (i.e. $N \to \infty$), it can be observed that these quantities tend towards infinite values. Taking into account the fact that $\psi_0 \to \infty$ when S_P tends towards 1, it can be deduced that both horizontal and vertical gradients of temperature and concentration are nearly zero within the enclosure. This implies that, in the vicinity of this particular value of S_P, uniform temperature and concentration fields are generated in the cavity. On the other hand and as a particular behaviour observed in the evolution of Sh, the horizontal gradient of concentration is also 0 ($Sh \to \infty$) for another value of S_P ($S_P \cong$ - 0.47) far from $S_P = 1$ as shown in Fig. (3). Finally, for the unstable branch, heat and mass transfer is dominated by the diffusive regime (Nu ≅ 1 and Sh ≅ 1/ (1-S_P)).

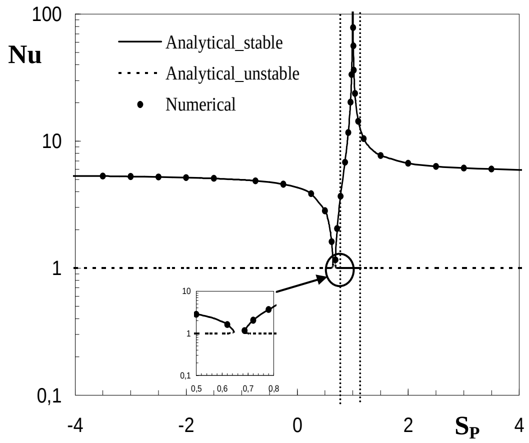

Figure 2: Variation of Nu vs. S_P illustrated for Le = 3, R_T = 100 and A_r = 12.

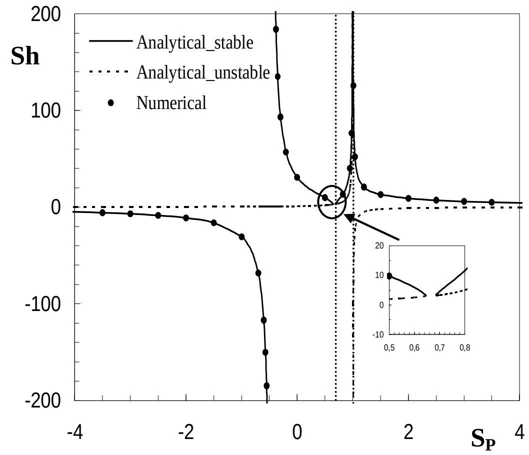

Figure 3: Variation of Sh vs. S_P illustrated for Le = 3, R_T = 100 and A_r = 12.

4. Conclusion

Fluid flow and heat and mass transfer induced by natural convection combined with Soret effect are studied analytically and numerically in a horizontal porous layer submitted, on its vertical short sides, to uniform heat and mass fluxes. The study focuses on the particular situation where the solutal to thermal buoyancy forces ratio is related to the Soret parameter by the relation $N = -1/(1 - S_P)$ for which the rest state is a solution of the problem. Only the sub-critical convection was found possible for this case and its threshold was determined analytically versus the governing parameters.

At sufficiently large values of R_T, it is demonstrated that the fourth order equation of ψ_0 has two solutions termed as "stable" and "unstable" and varying respectively as $R_T^{1/3}$ and R_T^{-1}.

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