Calculation of SIFs for semi-elliptical surface cracks in U-shaped bellows expansion joints of RR vessel

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Abstract

In the present study the problem of calculation of the stress intensity factors (SIFs) of semi-elliptical cracks located in the stress concentration areas of an expansion joint (EJ) situated between the lower barrel and the upper barrel of a pressure vessel research reactors(RRs) is numerically solved by advanced global-local finite element (FE) analysis. The common characteristic of the cases solved is that the stress field at the crack area varies along the axial, the circumferential, as well as, the through-the-thickness directions. SIF solutions for such problems are not available, neither analytically, nor numerically, as the currently existing solutions in the literature (numerical results, Newman–Raju empirical equations, weight function solutions, etc.) are only valid for uniform stress distribution along the axial and circumferential directions of the pressure vessel and allow variation only through-the-thickness. The crack locations considered are the intersection of the various parts that constitute the EJ and the connection of the EJ with the lower barrel. The stress intensity factors are presented in a suitable table format for various geometrical configurations of the semi-elliptical crack and of both the variation of a/c and a/t, thus providing a useful tool for the fracture mechanics design of cracked EJ. The modeling details of the methodology, employed in the analysis, are extensively discussed and the numerical approach is proven to be very efficient for the SIFs calculation of EJ semi-elliptical cracks.

Keywords: Stress intensity factor; Semi-elliptical surface crack; Expansion joint; Stress concentration

1. Introduction

Expansion joints as an element for compensating deformation are subjected to axial load from thermal deformation and often work under hostile environment (corrosion, radioactivity, etc.). Because some local regions of bellows with high elastoplastic strain undergo cyclic loadings caused by startup and shutdown and changeable operating conditions of RRs, they will be ruptured by crack growth. It is widely accepted that the existence of cracks in the metallic EJ, used in the nuclear domain, is unavoidable due to limitations of the manufacturing and welding processes (Fig. 1). The application of fracture mechanics principles in the design of EJ along with non-destructive inspection is the common way that engineers deal with this problem [1,2]. The standard procedure to evaluate the criticality of a surface crack in a thin walled cylindrical EJ for the case of brittle failure is by use of the well known stress intensity factor (SIF) formulas for semi-elliptical cracks [3,4].

Fracture analysis is widely used to predict component failure caused by preexisting small cracks, allowing one to take precautions to prevent further crack growth or to determine the remaining life of the structure. To obtain the fracture damage, stress intensity factors (SIFs) must be evaluated accurately. Because it is difficult to determine accurate SIFs using a closed-form analytical solution for cracks in complex structures, finite-element analysis is used instead.

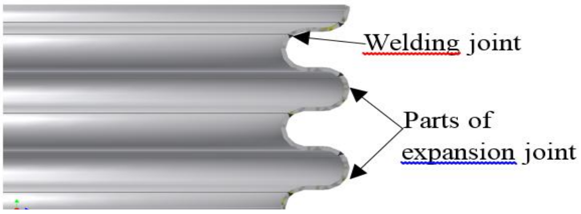
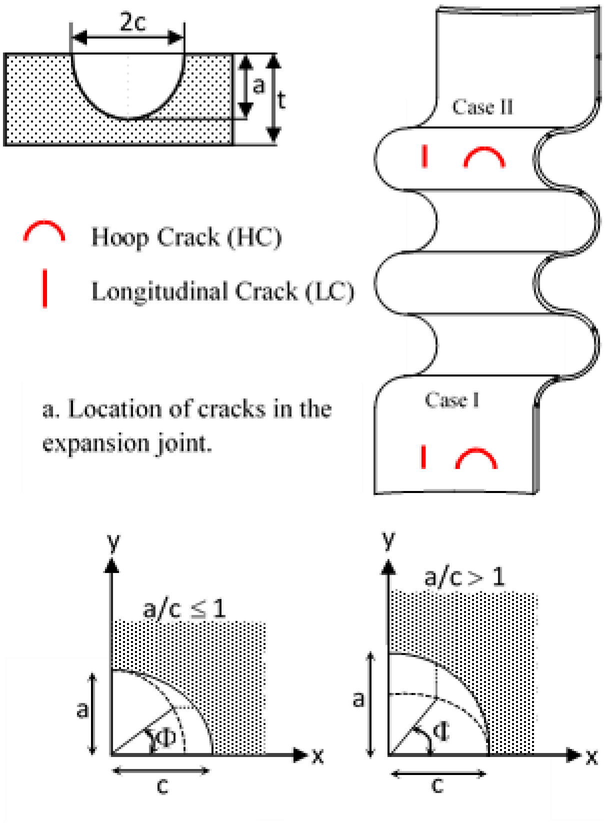


Fig. 1. Method of manufacture U-shape expansion joint and that welding processes.

Considerable work has been performed on the problem of semi-elliptical cracks contained in pressure vessels e.g. [6–10] but no more for EJ problem. The common topic of the studies is the investigation on the effect of the vessels radius to thickness ratio (t_c/R_c) and the effect of crack geometry ratio (a/c) on the stress intensity factor. However, the cracked pressure vessel problems that have been solved so far require that the stress field around the semi-elliptical crack varies only through-the-thickness of the vessel, but is constant along the axial and circumferential coordinates of the cylindrical shell. In fact the most susceptible areas for crack initiation are the areas of geometrical discontinuity, such as, the connection of the upper/lower barrel to the expansion joint, or the intersection area between the various parts that constitute the expansion joint, where stress concentrations occur and the stress field varies along the axial, as well as, depending on the considered load case, the circumferential direction.



a. Location of cracks in the expansion joint.

b. Coordinate system used to define parametric angle Φ.

Fig. 2. Schematic representation of the two cases examined in this study.

In the present study the finite element method is employed in order to calculate the SIFs of semi-elliptical cracks located around stress concentration areas. The examined cases are schematically shown in Fig. 2a. They all refer to a single semi-elliptical crack located at two different areas of the vessel. Case I (LC and HC) is the simple case, where the crack is sufficiently far away from the discontinuities (wiled joint between vessel and EJ) and has been studied for model development and verification purposes. In case II (LC and HC),the crack location is close to the expansion joint connection area. An ordinary FE approach for the problems under consideration would be inefficient, as the number of elements required for a three dimensional FE model of the whole vessel volume would be very high, leading to extremely time-consuming solutions, and therefore the methodology would not be appropriate for design purposes. To overcome this difficulty the FE sub-modeling technique has been applied. A numerical solution for the uncracked vessel without EJ is obtained initially, where the vessel been modeled without any crack. Consequently, the calculated displacement field is applied at the boundaries of sub-models representing the geometrical details of the cases shown in Fig. 2a, where cracks are included in the sub-models. Case I (LC) has been solved initially by the FE sub-modeling methodology and the obtained results for various ratios (t/R), and (a/c) are compared to the solutions given in Appendix D of the ASME Boiler and Pressure Vessel Code [4]. A good agreement has been observed, despite the differences in the numerical approaches employed. After the validation of the sub-modeling methodology, the method is applied to the calculation of SIFs for the case I and case II in the vessel with EJ (lower barrel, U-shape expansion joint and upper barrel). The crack geometries examined in all cases, cover a range of a/c (crack depth over crack length) ratios from 0.2 to 1, a/t(crack depth over wall thickness) ratios from 0.25 to 0.8 and $t_c/R_c = 0.034$ and 0.01.

2. Verification of the numerical approach

The hoop stress normalized along a longitudinal section of the vessel, as calculated from the initial pressure vessel FE model, in the absence of the semi-elliptical cracks. Normalization of the hoop stress has been performed according to its membrane component which equals to $\sigma_t = \frac{pR_m}{t_c} = 243\,MPa$ along the cylindrical section and $\sigma_t = \frac{pR_m}{2t_s} = 225\,MPa$ along the spherical section. It may be observed that as expected the two main stress concentration areas are the cylinder-to-spherical end connection due to the thickness reduction from $t_c = 50$ mm to $t_s = 27$ mm, as well as, the nozzle-to-cylinder intersection area due to the discontinuity in the pressure vessel body.

The problem of the stress intensity factor of a crack located at the internal vessel surface, away from the stress concentrations areas is solved initially, in order to validate the sub-modeling numerical approach. For verification purposes, the results are compared with those obtained from a different FE approach, described in Appendix D of Section VIII-Division 3 of the ASME code [4]. According to [4], the stress intensity factor may be calculated from Eq. (1):

$$K_1 = \begin{bmatrix} (A_0 + A_p)G_0 + A_1G_1 \\ +A_2G_2 + A_3G_3 \end{bmatrix} \sqrt{\pi a/Q} \qquad (1)$$

If the distribution of stress normal to the crack surface can be accurately represented by a single equation of the form of Eq. (2):

$$\sigma = A_0' + A_1'(x/t) + A_2'(x/t)^2 + A_3'(x/t)^3 \qquad (2)$$

For each value of a/t, the values of A_i' are converted to A_i values as follows:

$$A_0 = A_0' \,, A_1 = A_1'(a/t)\,, A_2 = A_2'(a/t)^2 \; and \; A_3 = A_3'(a/t)^3 \qquad (3)$$

Where G_i (i = 0–3) are the geometrical correction factors provided in [3], while Q is the elliptical integral given by:

$$Q = 1 + 4.593(a/2c)^{1.65} - q_y \qquad (4)$$

Moreover, q_y is the plastic zone correction factor calculated using the following equation:

$$q_y = \left[(A_0G_0 + A_1G_1 + A_2G_2 + A_3G_3)/S_y\right]^2/6 \qquad (5)$$

In Fig. 3, SIF results obtained from the FE sub-modeling approach, for the case I, are presented and compared to the SIF values calculated using Eqs. (1), (4) and (5). The SIF values shown in Fig. 3 refer to a crack parametric angle Φ = 90° (Fig. 2b), i.e. at the deepest point of the crack. A comparison between the results of the two approaches shows a good agreement, not exceeding a difference of 9%. This overall agreement observed gives confidence for the use of the present sub-modeling technique in the determination of SIF values of the other cases.

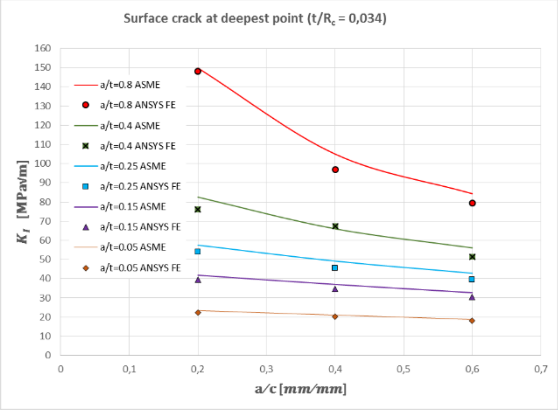


Fig. 3 Comparison of solutions

3. Numerical SIF results for semi-elliptical cracks in expansion joint of pressure vessel

In the present section, results are presented for all two cases considered (Fig. 2). The stress intensity factor distributions along the crack front for various surface cracks (a/c = 0.2–1.0) are presented in Figs. 9–25. The values obtained for pressure load are normalized according to the SIF value of a surface crack contained in an infinite plate, i.e. the numerically calculated SIF values K_I are divided by the term $\left(\frac{K_I}{\frac{pR_m}{t_c}\sqrt{\frac{\pi a}{Q}}}\right)$. The computedresults are also presented in Table format in Appendix A&B, for various a/c ratios in a form suitable for design purposes. The two cases are compared with each other in order to determine the most critical case.

4. Conclusions

In the present study the SIF solutions of a surface semi-elliptical crack located in different areas of stress concentration of a pressure vessel contained an expansion joint have been obtained using a FE-sub-modeling methodology. The numerical procedure is verified only for the case of a semi-elliptical crack located far away from stress concentration areas, by comparing the present methodology results to the available numerical results of the ASME code [3]. The modeling issues concerning the FE sub-modeling procedure of the present problem are discussed. The main conclusions drawn from the study can be summarized as follows:

- SIF results of cracks located in geometrical discontinuity areas of pressure vessels are computed and presented in the present work, for various a/c and a/t_c ratios in a form suitable for design purposes.
- The sub-modeling technique is proven to be an accurate and particularly efficient method for the determination of SIF values of semi-elliptical cracks in pressure vessels.
- The proposed methodology may be extended to arbitrary geometrical configurations of pressure vessels and variable loading conditions, as well as, to the solution of the problem of interaction between semielliptical cracks.

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