

Magnetic field effect on optimal separation of species in an inclined Darcy-Brinkman porous cavity saturated with an electrically conductive binary mixture

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Abstract

The present study deals with analytical and numerical investigation to examine the magnetic field effect on the species separation induced by combined effects of convection and Soret phenomenon in an inclined porous cavity saturated by an electrically conductive binary mixture. The porous medium is modeled according to Darcy-Brinkman's law and assumed to be homogeneous and isotropic. The relevant parameters for the problem are the thermal Rayleigh number ($R_r = 1$ to 10^6), the Lewis number ($Le = 10$), the inclination angle of the cavity ($\theta = 0^\circ$ to 180°), the separation parameter ($\varphi = 0.5$), the Darcy number ($Da = 10^{-5}$ to 10^3), the Hartmann number Ha ($Ha = 0$ to 100) and the aspect ratio of the cavity ($A_r = 12$). This study covers the limiting cases of Darcy and pure fluid media.

Key-Words: *Convection, porous medium, Soret effect, Species separation, magnetic field*

1. Introduction

The phenomenon of thermos-diffusion convection is behind the occurrence of concentration stratification in an initially homogeneous composition of mixtures. Owing to this phenomenon, thermosolutal separation is used for measurements of the diffusion coefficient, especially in liquid metals and fluids of industrial interest. Therefore, the more separation improvement in the experimental device, the more accurate and reliable data are obtained [1,2]. Recently, the subject of separation improvement in fluid mixtures has constituted a center of interest of numerous researchers. In this frame, some researcher teams [3] planned to carry experiments under microgravity conditions in order to minimize convection effect, which is responsible of the separation process disturbance. However, it is possible to improve the species separation in the presence of gravity by optimizing the coupling between convection and thermosolutal diffusion. To achieve this optimum coupling, Lorenz and Emery [4] proposed to fill the

cavity with a porous medium. In such a packed system, the maximum of separation is related to an optimal permeability of the porous medium. The so-called packed thermal diffusion cell described by these authors was used to perform experiments on varieties of ionic and organic mixtures [5-7]. Using the Darcy model, Rtibi et al. [8] studied the effect of a transverse magnetic field on buoyancy-driven convection in an inclined rectangular porous cavity, saturated with an electrically conducting mixture. It was reported in this study that the magnetic buoyancy force engenders a reduction of the flow intensity and heat transfer. However, its presence could engender an increase or a reduction of the mass transfer depending on the values of the Hartmann number, the inclination angle of the cavity and the separation ratio.

In this work the model was extended by using the Darcy-Brinkman to examine the porosity effect combined with the inclination angle of the cavity and the magnetic field on an eventual improvement of species separation.

2. Mathematical formulation

The system under investigation is a two-dimensional Darcy-Brinkman porous cavity of length L' and width H' , tilted at an angle θ with respect to the horizontal and bounded by four walls impermeable to mass transfer. The origin of the coordinate axes is taken at the center of the cavity. The long-side walls of the porous cavity are exposed to uniform fluxes of heat, q' , and transversal magnetic fields while its short walls are adiabatic. The fluid saturating the porous medium is assumed homogeneous, isotropic and modeled as a Boussinesq-incompressible fluid.

By neglecting the Dufour effect and using the vorticity-stream function formulation, the dimensionless governing equations in the stationary regime are presented as follows:

$$\nabla^2 \Psi + Ha^2 \frac{\partial u}{\partial y} = -R_T \left(\cos \theta \frac{\partial}{\partial x} - \sin \theta \frac{\partial}{\partial y} \right) (T + \varphi S) + Da \nabla^4 \Psi \quad (1)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \nabla^2 T \quad (2)$$

$$u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} = \frac{1}{Le} (\nabla^2 S - \nabla^2 T) \quad (3)$$

The velocity components are obtained as:

$$(u, v) = \left(\frac{\partial \Psi}{\partial y}, -\frac{\partial \Psi}{\partial x} \right) \quad (4)$$

In the above equations, Ψ , T and S are the dimensionless stream function, temperature and solute concentration, respectively. The boundary conditions associated to the governing equations are:

$$\left. \begin{aligned} y = \pm 1/2 : \Psi = \frac{\partial \Psi}{\partial y} = 0 \text{ and } \frac{\partial T}{\partial y} = \frac{\partial S}{\partial y} = -1 \\ x = \pm A_r/2 : \Psi = \frac{\partial \Psi}{\partial x} = 0 \text{ and } \frac{\partial T}{\partial x} = \frac{\partial S}{\partial x} = 0 \end{aligned} \right\} \quad (5)$$

In addition to the Lewis number, Le , the inclination θ of the cavity, the effective Darcy number, Da , and the cavity aspect ratio, A_r , the problem is governed by three other dimensionless parameters, namely, the separation parameter, φ , the Hartmann number, Ha and the thermal Darcy-Rayleigh number, R_T , which are defined as follows:

$$\left. \begin{aligned} \varphi = -\beta_S S'_0 (1 - S'_0) D_T / \beta_T D_{eff}, Da = \frac{\mu_e K}{\mu H^2} \\ Ha = B \sqrt{\gamma K / \mu}, A_r = L' / H' \\ R_T = g \beta_T \Delta T' K H' / \alpha v \end{aligned} \right\} \quad (6)$$

Where K is the permeability of the porous medium and D and D_T are respectively the mass diffusivity and the thermosolutal-diffusion coefficient.

The governing equations were solved analytically using the parallel flow concept and numerically with the finite difference method. Details of these methods are not given here due to the space limitation.

3. Results and discussions

For sparsely packed porous cavity ($Da = 0.01$) and $(\varphi, Le, R_T) = (0.5, 10, 200)$, the evolution of ΔC versus Ha illustrated by Fig.1, indicates that the separation phenomenon is negligible (case of $\theta = 45^\circ, 75^\circ, 90^\circ$ and 105°) or moderate and constant (cases of $\theta = 135^\circ$ and 150°) as long as $Ha \leq 2$. Beyond this limit of Ha ($2 < Ha \leq 100$), the separation undergoes an important increase towards a maximum whose value and location are nearly independent of θ within the range $[45^\circ, 105^\circ]$. For $\theta = 135^\circ$ and 150° , the location of the maximum of

separation is slightly shifted towards smaller values of Ha without any significant quantitative change. Substantial change in the evolution of the separation is observed for $\theta = 160^\circ, 163^\circ, 165^\circ$ and 170° . In fact, for $\theta = 165^\circ$, the separation of species is important in the range $0.1 \leq Ha \leq 1$ and undergoes a sharp decrease towards zero beyond $Ha = 2$. A similar behavior is observed for $\theta = 170^\circ$ but with less important separation of species. For $\theta = 160^\circ$ and 163° the evolution of the separation with Ha shows that the latter goes through a relative maximum (the maximum position is not very clear for $\theta = 163^\circ$) before undergoing a sharp decrease towards zero. For $Ha < 1$, the largest separation is obtained with $\theta = 163^\circ$ and 165° and it is nearly constant in this range of Ha (ΔC is around 0.49). Above $Ha = 12$, the maximum of separation is induced by varying θ in the range $[45^\circ, 105^\circ]$ with a very small change of the maximum position in this range. In the intermediate range $3 < Ha < 12$, the inclinations $\theta = 160^\circ, 150^\circ$ and 135° lead to maximum of separation for specific values of Ha in this range. More precisely, the maximum value of ΔC , which is about 0.49, is observed around $Ha = 5/7.4/10.5$ for $\theta = 160^\circ/150^\circ/135^\circ$, and around $Ha = 14.5$ for θ varying in the range $[45^\circ, 105^\circ]$. For $Ha < 3$, the largest uniform separation is obtained with $\theta = 163^\circ$ and 165° .

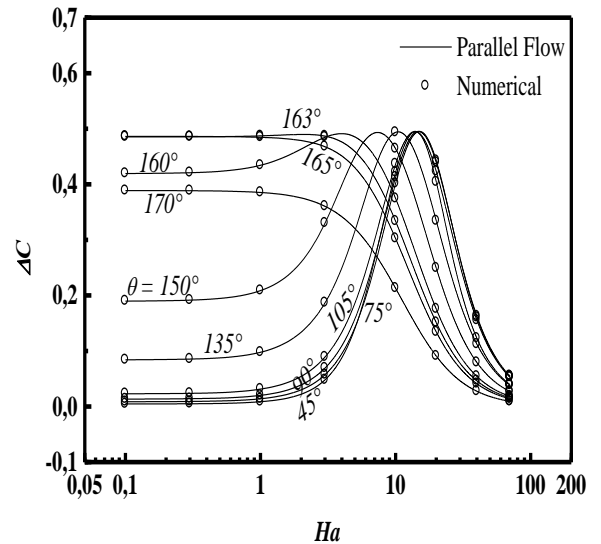


Fig.1: Effect of Ha on ΔC for $Le = 10$, $\varphi = 0.5$, $R_T = 200$ and $Da = 0.01$.

The iso-solutes illustrated in Fig.2 for $(\theta, Ha) = (163^\circ, 1)$, $(163^\circ, 30)$ and $(90^\circ, 14.75)$ show clearly that the separation of species depends strongly on the inclination and the magnetic field intensity when the values of the other parameters are fixed.

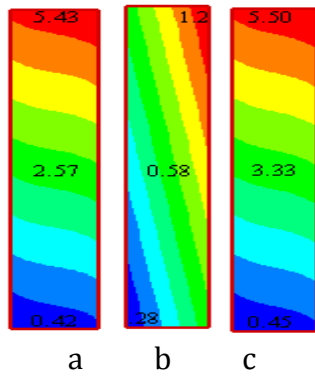


Fig.2: Iso-solutes lines for $Le = 10$, $\varphi = 0.5$, $R_T = 200$, $Da = 0.01$ and (a) $(\theta, Ha) = (163^\circ, 1)$, (b), $(\theta, Ha) = (163^\circ, 30)$ and (c) $(\theta, Ha) = (90^\circ, 14, 75^\circ)$.

The case of pure fluid medium, recovered with the Brinkman model for $Da = 10$, is illustrated in Fig.3 for $R_T = 10^5$. Compared to the Darcy medium, the evolution of ΔC versus Ha shows a big similarity. The curves show that the maximum separation magnitude allowed is nearly unchanged by changing the medium. However, the location of this maximum is strongly shifted towards larger values of Ha for θ varying in the range $45^\circ - 160^\circ$. In fact the increase of Ha (which has a damping effect on the flow) compensates the increase of the flow intensity caused by the increment of R_T . Subsequently, the flow intensity is brought back to the level ensuring an optimal separation. The threshold value of Ha , below which the inclinations $\theta = 163^\circ$ and 165° dominates in terms of maximum separation rises to a value close to $Ha = 59.5(Da, R_T) = (10, 10^5)$.

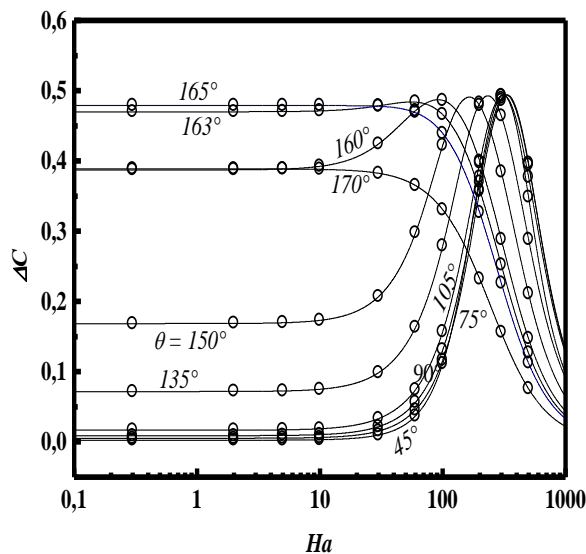


Fig.3: Effect of Ha on ΔC for $Le = 10$, $\varphi = 0.5$, $R_T = 10^5$ and $Da = 10$.

4. Conclusion

Illustrative results on the magnetic field effect on species separation in an inclined porous cavity saturated by an electrically conducting binary mixture in the presence of Soret effect are presented. The porous medium is modeled using Darcy-Brinkman extended model and the governing equations are solved analytically and numerically. The study is conducted in the case where the boundaries of the porous cavity are impermeable to mass transfer (pure Soret effect). The results presented cover the limits of weak and high values of the Darcy number. For $\varphi = 0.5$, the existence of specific ranges of Ha (depending on R_T , Da and θ) for which the separation reaches its maximum is proved.

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