Linear temporal and spatiotemporal analyses of instabilities in mixed convection of a viscoelastic fluid in a porous medium

P. Naderi[1], M.N. Ouarzazi[2], S.C. Hirata[2], H. Ben Hamed[1], H. Beji[1]

1. Laboratoire des Technologies Innovantes (LTI), Avenue des facultés, Amiens, France, pooya.naderi@etud.u-picardie.fr
2. Laboratoire de Mécanique de Lille, Lille, France

Abstract

The paper consists of the study of an elongated cavity filled with a viscoelestic fluid in a porous medium that is heated from below at a fixed temperature. If we have this system without flow it will experience natural convection and if the fluid flow exists, mixed convection rules over the system. In both cases the temperature difference creates different instabilities. At the present study we are interested in conducting calculations for Boger fluid that is considered a viscoelastic fluid with constant viscosity. Generally, the Oldroyd B model is used to describe the behavior of viscoelastic fluids. In the presence of fluid flow, Peclet number is a non-zero value and in this study the Peclet number is varied to check the influence of fluid flow on the instabilities. By the use of temporal and spatiotemporal methods to solve the equations the effect of the fluid flow on the weakly and highly viscoelastic zones has been checked. In addition, the effect of the relaxation time on the instabilities has been studied by comparison of results for different relaxation times. Hence, we fix Γ and vary Peclet number to plot the critical Rayleigh number curve as a function of Peclet number to see the effect of relaxation time on the instabilities and to compare the temporal and spatiotemporal methods simultaneously.

Keywords: *Viscoelastic Fluid, Porous medium, Mixed convection, Instabilities*

Nomenclature

Pe: Peclet number
Ra: Rayleigh number
K: Critical wave number
λ_1: Relaxation time
λ_2: Retardation time
Γ: Ratio of retardation time to relaxation time
ω: Critical frequency
μ_s: Newtonian solvent viscosity, $kg.m^{-1}.s^{-1}$
μ_p: Polymeric solute viscosity, $kg.m^{-1}.s^{-1}$
T_1^*: Upper plate temperature, K
T_0^*: Lower plate temperature, K
V: Velocity, $m.s^{-1}$
P: Pressure, $kg.m^{-1}.s^{-1}$

1. Introduction

The mixed convection in a system of a porous medium filled with a Newtonian fluid has been largely treated in previous research works. In addition, for a non Newtonian fluid especially a viscoelastic fluid like Boger fluid, the natural convection has already been studied. However, in this study the forced convection of a viscoelastic fluid in a porous medium between two horizontal plates have been examined. The lower plate has a fixed higher temperature and the upper plate is maintained at a lower temperature. The heat together with the flow creates different instabilities in the system. The practical application of this kind of research is in the extrusion of polymer fluids, solidification of liquid crystals, suspension solutions and petroleum activities [1]. The Peclet number Pe is defined by the strength of fluid flow and the dimensionless Rayleigh number Ra is defined by the temperature difference across the layer. In rheology models for viscoelastic fluids besides Ra there are two other dimensionless numbers as the relaxation number, which defines the stress relaxation time, and the retardation number, which is the strain retardation time [1]. Boger fluids are viscoelastic fluids with a constant viscosity. In this case, due to the independence of viscosity from shear rate, elastic effects can be separated from viscous effects in viscoelastic flows as the latter effects can be determined with Newtonian fluids. Boger fluids are dilute polymer solutions generally made with a solvent sufficiently viscous that stresses due to elasticity are measurable [2]. Therefore it is interesting to take Boger Fluid (Γ=0.75) as an example of viscoelastic fluids to conduct the calculations.

2. Mathematical formulation of the system

We consider a porous medium between two infinite plates saturated by a viscoelastic fluid obeying the Oldroyd B law. The height between the plates is H and the temperatures at the lower and upper plates are maintained respectively at T_0^* and T_1^*. The temperatures are fixed in a way that $T_1^* < T_0^*$. Moreover, there is a horizontal flow with the velocity V imposed to the system. With regards to the Boussinesq approximation and Darcy law for viscoelastic fluids with some replacements and by putting the equations in the dimensionless order like the steps done in the work done by S.C Hirata et al. [3], the dimensionless equations are written as:

$$\nabla . V = 0 \quad (1)$$
$$(1 + \Gamma\lambda_1 \partial_t)V + (1 + \lambda_1\partial_t)(\nabla P - RaT\mathbf{e}_z) = 0 \quad (2)$$
$$\partial_t T + V.\nabla T - \nabla^2 T = 0. \quad (3)$$

The plates are considered impermeable and maintained at fix temperatures, therefore the conditions are written as:

$$T(x, y, z = 0) = 1, T(x, y, z = 1) = 0$$
$$and\ V.\mathbf{e}_z = 0\ at\ z = 0, 1. \quad (4)$$

There is also a uniform horizontal fluid flow which is represented by:

$$V.\mathbf{e}_x = Pe. \quad (5)$$

There are four dimensionless numbers that affect the system: the Rayleigh number (Ra), Peclet number (Pe), relaxation time (λ_1) and retardation time (λ_2) [3]. In this study, the major focus is more put on fixing the ratio of the retardation time and the relaxation time (Γ) that is defined as:

$$\Gamma = \frac{\lambda_2}{\lambda_1} = \frac{\mu_s}{\mu_s + \mu_p}. \quad (6)$$

Where μ_s is the Newtonian solvent viscosity and μ_p is the polymeric solute viscosity because viscoelastic fluids and especially Boger fluids are fabricated by adding a small amount of polymer to a Newtonian fluid solvent.

Therefore:

$$\lambda_2 = \Gamma\ \lambda_1. \quad (7)$$

Thus, the equations are written as a function of Ra, Pe, λ_1 and Γ like in equation (2).

The use of linear stability analysis of the stationary conduction solution results in a system of partial differential equations validated by temperature, pressure and velocity fluctuations [3]. A normal modes analysis for checking the boundary conditions leads us to the dispersion equation:

$$D(\omega, k, \lambda_1, \lambda_2, Ra, Pe) = \omega^2\lambda_2(k^2 + \pi^2) + (k^2 + \pi^2)(-ik_xPe - k^2 - \pi^2) + k^2Ra + i\omega(k^2 + \pi^2 + \lambda_2(k^2 + \pi^2)(ik_xPe) + \lambda_2(k^2 + \pi^2)^2 - Rak^2\lambda_1) = 0 \quad (8)$$

Where $k^2=k_x^2+k_y^2$ and k_x is the wave number in the flow direction and k_y is the wave number in transversal direction [3]. For natural convection, Pe=0, the solution for stationary instability is obtained by imposing a zero frequency and the solution for oscillatory instability is provided by assuming a zero imaginary frequency. By solving the dispersion equation for a none-zero Peclet number (the case with fluid flow) with a real wave number and a complex frequency it is achieved to study the behavior of the system in two extreme zones (weakly and highly viscoelastic zones).

In temporal analysis the system is imposed a real wave number and a complex frequency. On the contrary, in spatiotemporal analysis the frequency is a real and the wave number is a complex number.

3. Identifying the weakly and highly viscoelastic zones at Pe = 0

The curve of λ1 according to Γ from the equation $Ra_C^O = Ra_C^S = 4\pi^2$ must be plotted in order to distinguish the two zones at Peclet = 0. The equation is as shown below and the curve is shown in Figure 1.

$$\frac{1}{\lambda 1}\left(1 + 2\lambda 1\Gamma\pi\left(\pi + \sqrt{\pi^2 + \frac{1}{\lambda 1\Gamma}}\right)\right) = 4\pi^2. \quad (9)$$

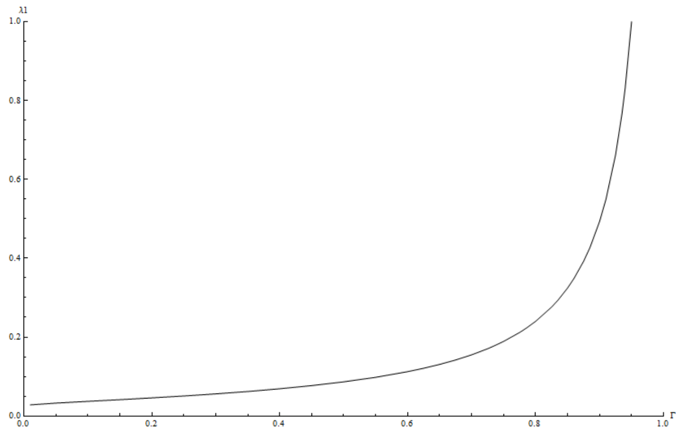

Figure 1 Zone distinction graph

This graph shows the three important behaviors of the fluid in this system including the weakly viscoelastic zone, which is the area under the curve, the threshold (the curve) and the highly viscoelastic zone, which is the area above the threshold curve. In this study we focus mostly on the two extreme cases of weakly and highly viscoelastic zones.

4. Results and Conclusions

In order to compare the convective and absolute instability methods (respectively the temporal and spatiotemporal analyses) two same series of calculations are conducted with a fixed Γ=0.75 and different relaxation times (λ_1). Furthermore, both weakly viscoelastic and highly viscoelastic zones have been covered in the calculations. The results in the weakly viscoelastic zone consist a fixed Γ=0.75 and different relaxation times as λ_1=0.01, λ_1=0.03 and λ_1=0.05 for both temporal and spatiotemporal analyses as illustrated in the figure 2. In the same way the results for the highly viscoelastic zone include a fixed Γ=0.75 and different relaxation times as λ_1=0.6, λ_1=0.7 and λ_1=0.9 for both temporal and spatiotemporal analyses as shown in Figure 3. The Figures 2 and 3 show the results for Peclet numbers up to 10. In order to see clearly the difference between the curves, figures 4 and 5 illustrate a zoom for Peclet numbers up to three.

It should be noted that in the weakly viscoelastic zone there is only one mode (one solution), which is evident in figure 2, and there are three modes (three solutions) in the highly viscoelastic zone. However, in our figures there is just one mode represented for highly viscoelastic zone, which shows the most unstable mode among the three modes.

The first noticeable point from both Figures 2 and 3 is the destabilizing effect of relaxation time on the system. In both weakly and highly viscoelastic zones as the relaxation time increases, the critical Rayleigh number decreases. The decrease in the critical Rayleigh number results in a more unstable and chaotic system.

The second important point is the difference between temporal and spatiotemporal analyses which is small for low Peclet numbers and larger for high Peclet numbers. For instance, the Rayleigh number for Pe=0 is the same as the stationary solution and for all values of relaxation time it does not change as in Figures 2 and 4. Considering the highly viscoelastic zone Figures (3 and 5), the values of Rayleigh number for Pe=0 are not the same and even a decrease is observed for the equivalent values in temporal analysis. Therefore, the results for temporal analysis are more unstable than the results for spatiotemporal analysis. On the one hand, this difference between the results from the two methods increases as Peclet number increases, as it can clearly be seen in Figures 2 to 5. Moreover, in spatiotemporal analysis the tendency is upward (less chaotic) with Peclet number and conversely in temporal analysis the tendency is downward (more chaotic) with Peclet number.

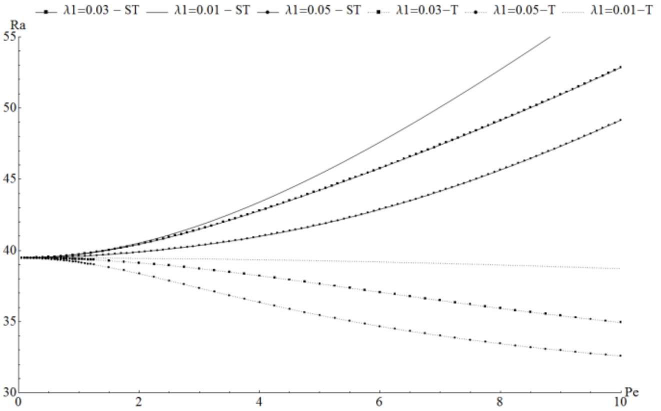

Figure 2 Comparison of results from temporal and spatiotemporal analyses as well as the effect of relaxation time on the instabilities for a fixed Γ = 0.75 in the weakly viscoelastic zone

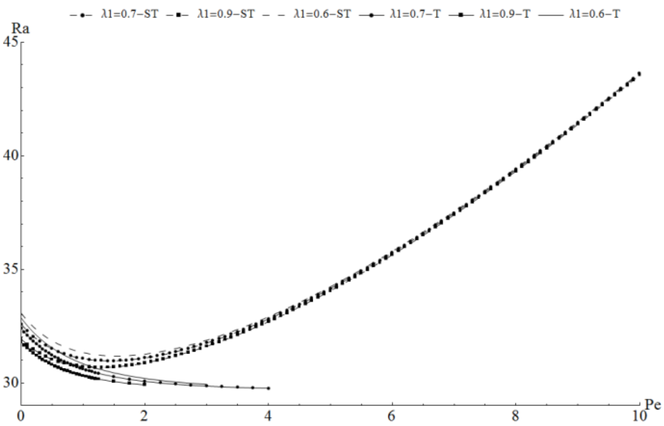

Figure 3 Comparison of results from temporal and spatiotemporal analyses as well as the effect of relaxation time on the instabilities for a fixed Γ = 0.75 in the highly viscoelastic zone

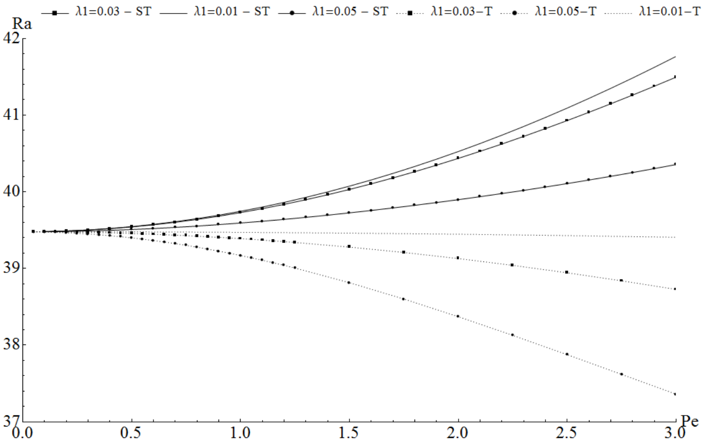

Figure 4 Comparison of results from temporal and spatiotemporal analyses as well as the effect of relaxation time on the instabilities for a fixed Γ = 0.75 in the weakly viscoelastic zone for Peclet numbers up to 3

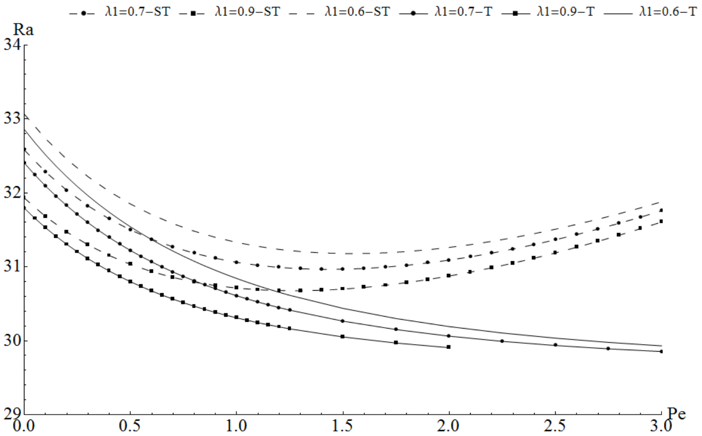

Figure 5 Comparison of results from temporal and spatiotemporal analyses as well as the effect of relaxation time on the instabilities for a fixed Γ = 0.75 in the highly viscoelastic zone for Peclet numbers up to 3

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