Analysis and Dynamic Modeling of Planar Parallel Robot

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Abstract

Parallel robots have obtained a big attention because of their great advantages more than serial mechanism, like high stiffness, high speed and high accuracy. Parallel robots have also some drawbacks, the major of them are a limited workspace that can be seen as the intersection of the individual workspace of each serial arm and also the complicated singularities.Different methods were used to describe the problem and achieve the dynamics modeling of the robot Thus, there is still no consensus on which formulation is the best to describe the dynamics of the robot. Lagrange formulation was used in this work because it offers a well analytical and orderly structure. In order to verify the validity of the dynamic model proposed in this study, simulations of the obtained model are carried out using Matlab software.

The study of the dynamics of the 3 dof parallel robot is made mostly to solve successfully the control of the motion of this robotic system.

Keywords:Dynamics Modeling,Jacobian Matrix, Lagrange formulation, Matlab software, Parallel robot

1. INTRODUCTION

In recent years, parallel robots have received huge attention for different researches and applications. Parallel manipulators are closed-loop mechanisms in which the separate serial chains (links and joints) are connected to both the fixed base and the moving platform. This new type of robots can achieve certain tasks that serial robots are not able to perform. Even though serial robots have advantages like simple dynamic and kinematic models, large workspace, but parallel robots have advantages more potential than serial robots as better stiffness, high speed, higher kinematical precision, large load carrying capacity and the most important is the possibility of fixing actuators on the base, which can be used for fast and precise operations. Parallel robots have also some drawbacks, the major of them are a limited workspace that can be seen as the intersection of the individual workspace of each serial arm and also the complicated singularities.

Parallel manipulators can be equipped with revolute or prismatic actuators. They have a strong construction and they are able to move bodies of large dimensions with high velocities and accelerations [1]. Parallel manipulators can be divided into two fundamental categories, namely spatial and planar manipulators, The first category composes of the spatial parallel manipulators that can translate and rotate in the three dimensional space, this category recognized as Stewart–Gough platform, has received great attention [2-3]. They are used in flight simulators.

The planar parallel manipulator which composes of second category, translate along the x and y-axes, and rotate around the z-axis, they are useful for manipulating an object on a plane. Liu Shanzeng[5] which present a kinematical study of a planar parallel robot, where a moving platform is connected to a fixed base by three links, each leg consisting of two binary links and three parallel revolute joints. Many researchers give an interesting numerical solution in the inverse and direct kinematics of this kind of planar robot.

Complex kinematics and dynamics frequently guide to model simplifications decreasing the accuracy. In order to surmount this problem, accurate kinematic and dynamic identification is required. The kinematic and dynamic modeling of 3RRR is extremely difficult in comparison with serial robots.

In the literature, there are different approaches to develop the dynamic model of parallel robots: the Lagrangian Formulation, the application of Newton–Euler laws and the principle of Virtual Work. Generally, these approaches calculate the dynamic model in terms of the active joints. It should be noted that, in the general case, numerical methods are required to resolve it due to the highly nonlinear kinematic equations of the mechanism, which increase the computational cost of the model.

In this work we present a dynamic model of 3 RRR parallel robot using Lagrange approach in order to solve successfully the control of the motion of this robotic system.

Kinematic Description of Planar Parallel robot :

Description of 3 RRR Parallel Robot:

This work presented a geometric description of 3 RRR parallel robots which composed by a mobile platform (MP) and three RRR serial chains that join it to a fixed base (Fig. 1). Each RRR chain is a serial chain composed by three rotational R joints.

Allow P(x, y) be the end-effectorposition in the plane and φ its orientation. Let O be the origin of the fixed reference frame and let $A_i, B_i \ and \ C_i$ with $i = 1, 2, 3$ define the rotational articulations of each leg. A_i Points are actuated, so that the actuators are fixed to the base. l_{A_i} is the length

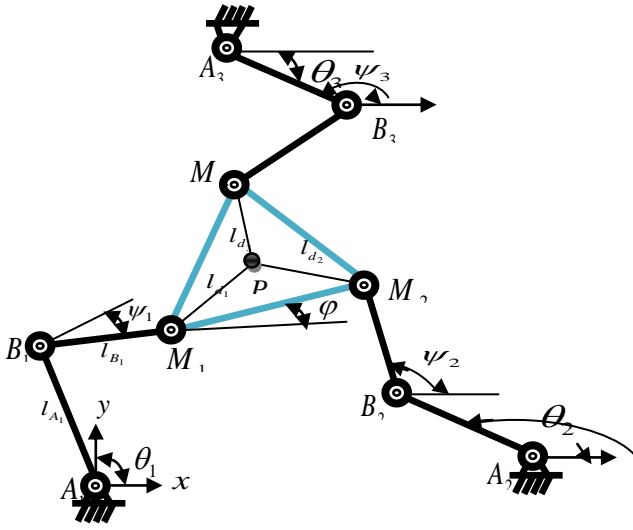of Link A, l_{B_i} is the length of Link B.

Fig.1: geometric description of 3 RRR parallel robot

Inverse Kinematic study:

The loop closure equation for the 3-RRR parallel robot is as follow[4]:

$$OP = OA_i + A_iB_i + B_iM_i + M_iP \qquad (1)$$

Writing the equations in component form:

$$
\begin{aligned}
P_x &= l_{Ai}\cos(\theta_i) + l_{Bi}\cos(\psi_i) + l_{di}\cos(\sigma_i + \varphi) + x_{Ai} \\
P_y &= l_{Ai}\sin(\theta_i) + l_{Bi}\sin(\psi_i) + l_{di}\sin(\sigma_i + \varphi) + y_{Ai}
\end{aligned} \qquad (2)
$$

Where

σ_i angle between the line from l_{di} joint P to the joint at M_i and it is considered from the line between

$M_1 \, and \, M_2$ on the moving platform..

To compute the inverse kinematics, we can rewrite the equation as follows:

$$K_i\sin\theta + F_i\cos\theta = E_i \qquad (3)$$

Where :

$$
E_i = P_x^2 + P_y^2 + l_{Ai}^2 + l_{Bi}^2 + l_{di}^2 + x_{Ai}^2 + y_{Ai}^2 - 2P_x l_{di}\cos(\sigma_i + \varphi) - 2P_x x_{Ai}
$$
$$
+ 2l_{di}x_{Ai}\cos(\sigma_i + \varphi) - 2P_y l_{di}\sin(\sigma_i + \varphi) - 2P_y y_{Ai} + 2l_{di}y_{Ai}\sin(\sigma_i + \varphi)
$$

$$F_i = -2l_{Ai} + 2l_{Ai}l_{di}\cos(\sigma_i + \varphi) + 2l_{Ai}x_{Ai}$$

$$K_i = -\left[-2P_y l_{Ai} + 2l_{Ai}l_{di}\sin(\sigma_i + \varphi) + 2l_{Ai}y_{Ai}\right]$$

The inverse kinematics solution for equation (3) is:

$$\theta_i = A\tan 2(K_i, F_i) \pm A\tan 2(\sqrt{(K_i^2 + F_i^2 - E_i^2)}, E_i) \qquad (4)$$

Where:
$$i = 1, 2, 3$$

Inverse Jacobian Matrix of the robot

Many methods are defined to find the Jacobian matrix of the parallel mechanism including differentiation of the inverse kinematic equation. The Matrix Jacobian is defined as the matrix representing the transformation mapping the joint rates into the Cartesian velocities transformation

Thus, by differentiating the equation (1) we can obtain the following equation:

$$J_x\dot{X} = J_\theta\dot{\Theta}$$

$$
J_x = \begin{bmatrix}
\cos(\lambda_1) & \sin(\lambda_1) & -l_{d_1}\sin(\lambda_1 - (\sigma_1 + \varphi)) \\
\cos(\lambda_2) & \sin(\lambda_2) & -l_{d_2}\sin(\lambda_2 - (\sigma_2 + \varphi)) \\
\cos(\lambda_3) & \sin(\lambda_3) & -l_{d_3}\sin(\lambda_3 - (\sigma_3 + \varphi))
\end{bmatrix} \text{ et } J_\theta = \begin{bmatrix}
l_{A_1}\sin\psi_1 & 0 & 0 \\
0 & l_{A_2}\sin\psi_2 & 0 \\
0 & 0 & l_{A_3}\sin\psi_3
\end{bmatrix}
$$

Avec

$$\lambda = \theta_i + \psi_i$$

So, $\quad J = J_\theta^{-1}J_x$

$$
J = \begin{bmatrix}
\dfrac{\cos(\lambda_1)}{l_{A_1}\sin\psi_1} & \dfrac{\sin(\lambda_1)}{l_{A_1}\sin\psi_1} & \dfrac{-l_{d_1}\sin(\lambda_1 - (\sigma_1 + \varphi))}{l_{A_1}\sin\psi_1} \\[2ex]
\dfrac{\cos(\lambda_2)}{l_{A_2}\sin\psi_2} & \dfrac{\sin(\lambda_2)}{l_{A_2}\sin\psi_2} & \dfrac{-l_{d_2}\sin(\lambda_2 - (\sigma_2 + \varphi))}{l_{A_2}\sin\psi_2} \\[2ex]
\dfrac{\cos(\lambda_3)}{l_{A_3}\sin\psi_2} & \dfrac{\sin(\lambda_3)}{l_{A_3}\sin\psi_2} & \dfrac{-l_{d_3}\sin(\lambda_3 - (\sigma_3 + \varphi))}{l_{A_3}\sin\psi_3}
\end{bmatrix}
$$

Dynamics modeling:

In this section, we investigate the dynamics of the robot and derive the dynamic model of the mechanism basedon the kinematic modeling which was written in previoussection.

To calculate the dynamic model of the robot, the Lagrangianapproach is used, the generalized coordinates required todescribe thesystemas follows:

$$\tilde{q} = \begin{bmatrix} x_p & y_p & \varphi & \theta_1 & \theta_2 & \theta_3 \end{bmatrix}$$

The general formulation of Lagrangianfunctionis defined as

$$L(q,\dot{q}) = K(q,\dot{q}) - U(q) \qquad (6)$$

Where:

$K$is the kinetic andU represent the potential energy of therobot. Then, the motion equations of the robot can be derived written as

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{q}} - \frac{\partial K}{\partial q} + \frac{\partial U}{\partial q} = \tau \qquad (7)$$

Where: τ is a vector of torques of actuators.

Thus, The potential energy is zero because the robot moves in horizontal plane, Therefore, the Lagrangian function is simply equals to the total kinetic energy of the robot. The kinetic energy of the manipulator can be divided into three parts; the kinetic energy of the first link in each limb, the second link in each limb, and the moving platform.

The kinetic energy of the mechanism can bewritten as

$$K(q,\dot{q}) = K_p + \sum_{i=1}^{3} K_{l_{Ai}} + \sum_{i=1}^{3} K_{l_{Bi}}$$

Where: K_p, $K_{l_{A_i}}$ and $K_{l_{A_i}}$ are the kinetic energy of the platform and the first and the second leg of the robot.

Thus, the dynamic model of the robot is written as follows:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau \qquad (7)$$

Where:

$q, \dot{q}, \ddot{q} \in \square^n$ denotes the joint angle, the joint velocity and the joint acceleration, $M(q) \in \square^{n \times n}$ is the manipulator inertia matrix, $C(q,\dot{q}) \in \square^{n \times n}$ is the centrifugal and Coriolis force matrix, $G(q) \in \square^n$ is the gravitational force vector and $\tau(t)$ denotes the torque.

Dynamic model using SimMechanics:
In order to simulate model of the robot, the dynamic model of the structure must be built.

In this work, the dynamic model of the robot is built using the SimMechanicstoolbox from Simulink. The model is built using Simulink blocks, which represents the kinematic elements and the joints of the robot.

The model of the mechanical structure is presented in fig.3. All three kinematic chains are defined by body and joint blocks.
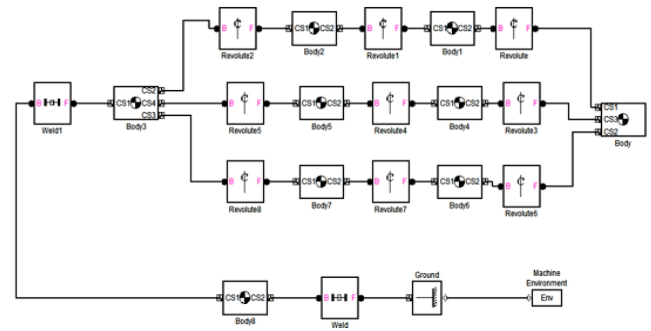

Fig.2 model of the mechanical structure

Conclusion:

In thisworkwepresentanapproachfordeterminingtheinversekinematics and thedynamicsmodelappliedon 3-RRR planar parallel. Planar parallel robots haveimportantpotentialfordifferentapplications. thedymanicsmodelwhichiscomplicateddduetoexistence of multipleclosed-loopchains. Thus a greatdynamic performance and a higherprisicion in positioning of themovingplatformunderimportant load are required, thedynamicmodel of the robot alsoisrequiredfortheautomatic control.

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