# Numerical Study of the Effect of a Vertical Magnetic Field on the Onset of Natural Convection in a Porous Medium Saturated By a Nanofluid

**Abderrahim Wakif** *, Zoubair Boulahia,Wafaa Bentoto , and Rachid Sehaqui
Hassan II University, Faculty of Sciences Aïn Chock, Laboratory of Mechanics, B.P.5366 Mâarif, Casablanca, Morocco.
wakif.abderrahim@gmail.com

## Abstract

The effect of a vertical magnetic field on the onset of convection in an electrically conducting nanofluid layer heated from below is investigated using the linear stability theory and the Buongiorno's mathematical model. In this investigation, we consider that the nanofluid is Newtonian and confined in a porous medium. The eigenvalue problem obtained is solved numerically for rigid-rigid boundaries using the Chebyshev-Gauss-Lobatto method.

**Keywords:** *Linear stability, Nanofluid, Magnetic field, Spectral method.*

## 1. Introduction

Our work consists of studying the Rayleigh-Bénard problem in a Darcy-Brinkman porous medium saturated by an electrically conducting nanofluid layer and excited by a uniform vertical magnetic field in the rigid-rigid case where the volumetric fraction of the nanoparticles at the top wall is considered as greater than that of the bottom.

The problem studied will be solved with a more accurate numerical method based on Chebyshev-Gauss-Lobatto technique. In this investigation we assume that the nanofluid is Newtonian and the parameters which appear in the governing equations are considered constant in the vicinity of the temperature of the cold wall $T_c^*$ which we took it as a reference temperature. Finally we will impose that the flow is laminar and the radiation heat transfer mode between the horizontal walls will be negligible compared to other modes of heat transfer.

To show the accuracy of our method in this study, we will check some results treated by D.Yadav et al. [1], concerning the study of the convective instability of a nanofluid (Water + Alumina) in presence of a uniform vertical magnetic field .Our numerical method is used in this investigation to give results with an absolute error of the order of $10^{-5}$ to the exact critical values characterizing the onset of the convection.

## 2. Mathematical Formulation

We consider an infinite horizontal layer of incompressible electrically conducting nanofluid saturated a porous medium, this layer is considered of thickness $L$ , heated from below and confined between two parallel impermeable boundaries, where $T_h^*$ , $T_c^*$ are the temperatures and $\chi_h^*$ , $\chi_c^*$ are the nanoparticle's volume fractions at lower and upper boundaries, respectively. The nanofluid layer is subjected to a uniform vertical magnetic field, such that the gravity field $\vec{g}$ is acting vertically downwards and the magnetic field $\vec{H}^*$ is acting in vertically upwards direction (Fig.1).

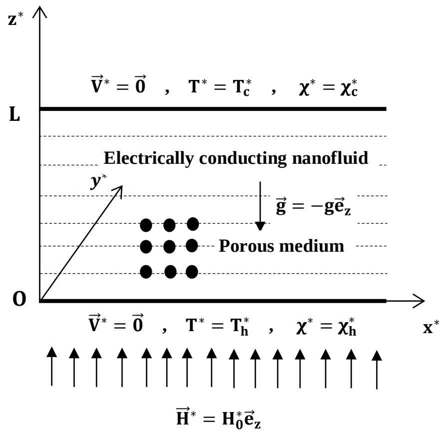


**Fig.1.** Physical system and geometry

Using the Boussinesq approximation, the relevant basic expressions of the mass, momentum, thermal energy, nanoparticles and Maxwell's equations in a Darcy-Brinkman porous medium are [1-6]:

$$\vec{\nabla}^*.\vec{V}^* = 0 \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\left[\frac{\partial}{\partial t^*} + \frac{1}{\varepsilon}\left(\vec{V}^*.\vec{\nabla}^*\right)\right]\vec{V}^* = -\vec{\nabla}^* P^* + \left(\tilde{\mu}\vec{\nabla}^{*2} - \frac{\mu}{K}\right)\vec{V}^* + \rho\vec{g} + \frac{\mu_e}{4\pi}\left(\vec{\nabla}^* \times \vec{H}^*\right) \times \vec{H}^* \tag{2}$$

$$\rho = \rho_p \chi^* + \rho_0[1 - \beta(T^* - T_c^*)](1 - \chi^*) \tag{3}$$

$$\left[\sigma\frac{\partial}{\partial t^*} + \left(\vec{V}^*.\vec{\nabla}^*\right)\right]T^* = \alpha_m \vec{\nabla}^{*2} T^* + \varepsilon D_B \frac{(\rho c)_p}{(\rho c)_f}\vec{\nabla}^*\chi^*.\vec{\nabla}^* T^* + \varepsilon D_T \frac{(\rho c)_p}{T_c^*(\rho c)_f}\vec{\nabla}^* T^*.\vec{\nabla}^* T^* \tag{4}$$

$$\left[\frac{\partial}{\partial t^*} + \frac{1}{\varepsilon}\left(\vec{V}^*.\vec{\nabla}^*\right)\right]\chi^* = D_B \vec{\nabla}^{*2}\chi^* + \left(\frac{D_T}{T_c^*}\right)\vec{\nabla}^{*2} T^* \tag{5}$$

$$\vec{\nabla}^*.\vec{H}^* = 0 \tag{6}$$

$$\left[\frac{\partial}{\partial t^*} + \frac{1}{\varepsilon}\left(\vec{V}^*.\vec{\nabla}^*\right)\right]\vec{H}^* = \frac{1}{\varepsilon}\left(\vec{H}^*.\vec{\nabla}^*\right)\vec{V}^* + \eta\vec{\nabla}^{*2}\vec{H}^* \tag{7}$$

The boundary conditions applied on the walls of the system are:

$$\begin{cases} w^* = 0 , \dfrac{\partial w^*}{\partial z^*} = 0 , T^* = T_h^* , \chi^* = \chi_h^* & \text{at} \quad z^* = 0 \\ w^* = 0 , \dfrac{\partial w^*}{\partial z^*} = 0 , T^* = T_c^* , \chi^* = \chi_c^* & \text{at} \quad z^* = L \end{cases} \tag{8}$$

## 3. Numerical Method and Validation

Considering the dimensionless forms of the governing Eqs. (1)- (7), disturbing the basic solutions, linearizing the equations, eliminating the pressure from the momentum equation by operating curl twice and retaining the vertical component and analyzing the disturbances into normal modes , the linear stability equations of the stationary mode can be matrically formulated as follows:

$$\mathbf{A\,X} = \mathrm{R_a}\,\mathbf{B\,X} \qquad (9)$$

The linear stability theory shows that the oscillatory mode of heat transfer is not possible via the free.free case in the case where $\chi_h^* < \chi_c^*$ and the stationary stability depends on the modified magnetic Chandrasekhar number $\mathrm{Q}$ , the modified specific heat increment $\mathrm{N_B}$ , the modified Lewis number $\mathrm{L_e}$ , the nanoparticle Rayleigh number $\mathrm{R_N}$ , the modified diffusivity ratio $\mathrm{N_A}$ and the Darcy number $\mathrm{D_a}$ .

The eigenvalue problem (9) with the boundary conditions (8) is solved numerically using the Chebyshev spectral method where the thermal Rayleigh number $\mathrm{R_a}$ is its corresponding eigenvalue. To have a check on the accuracy of the numerical procedure used in this study, we compare our results with those of D.Yadav et al. [1] in Table.1 for a nanofluid (e.g. $Al_2O_3 + H_2O$) characterized by $\mathrm{N_B} = 7.5 \times 10^{-4}$, $\mathrm{L_e} = 5000$, $\mathrm{R_N} = 0.122$ and $\mathrm{N_A} = 1$ and confined in a non-porous medium.

**Table.1.** comparison of the present results with yadav's results for Alumina-Water nanofluid in the case where $\mathrm{N_B} = 0.00075$, $\mathrm{R_N} = 0.1$, $\mathrm{L_e} = 5000$ and $\mathrm{N_A} = 5$

| Q | D .Yadav et al. [1] | | Present study | |
|---|---|---|---|---|
| | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ |
| 0 | 3.136 | 1097.640 | 3.1202 | 1097.2500 |
| 100 | 4.012 | 3147.108 | 4.0119 | 3147.1081 |
| 200 | 4.446 | 4878.418 | 4.4457 | 4878.4112 |

## 4. Results and Discussion

The effect of Lorentz forces due to the presence of a uniform vertical magnetic field on the criterion of the onset of thermal convection in an incompressible electrically conducting nanofluid saturated a porous medium is investigated.

The principal results derived from the present analysis can be summarized as follows:

- An increase in the modified Lewis number $\mathrm{L_e}$ , the nanoparticle Rayleigh number $\mathrm{R_N}$ or the modified diffusivity ratio $\mathrm{N_A}$ allows to destabilize the nanofluids.
- The Chandrasekhar number $\mathrm{Q}$ and the Darcy number $\mathrm{D_a}$ have a stabilizing effect on the stationary convection.
- The modified particle-density increment $\mathrm{N_B}$ has no significant effect on the convective instability.
- The presence of Lorentz forces allows to reduce the size of convection cells (see Fig.2).
- An increase in the Darcy number $\mathrm{D_a}$ allows to increase the size of convection cells (see Fig. 3).
- The parameters $\mathrm{N_B}$, $\mathrm{L_e}$ , $\mathrm{R_N}$ and $\mathrm{N_A}$ have no effect on the size of convection cells.

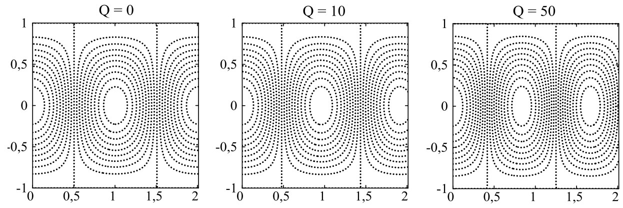


**Fig.2.** Streamlines at various modified magnetic Chandrasekhar numbers $\mathrm{Q}$ in the case where $\mathrm{N_B} = 0.01$, $\mathrm{R_N} = 0.1$, $\mathrm{L_e} = 1000$ , $\mathrm{N_A} = 1$ and $\mathrm{D_a} = 0.8$

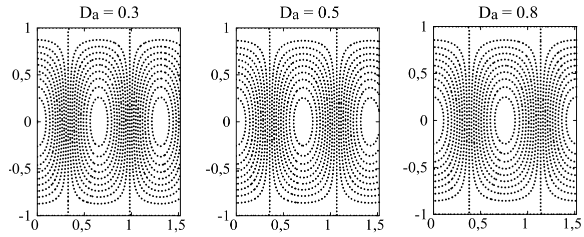


**Fig.3.** Streamlines at various Darcy numbers $\mathrm{D_a}$ in the case where $\mathrm{Q} = 100$, $\mathrm{N_B} = 0.01$, $\mathrm{R_N} = 0.1$, $\mathrm{L_e} = 1000$ and $\mathrm{N_A} = 1$

## References

[1] Dhananjay Yadav, R. Bhargava and G.S. Agrawal, “Thermal instability in a nanofluid layer with a vertical magnetic field,” J .Eng. Math, vol.80, pp.147-164, 2013.

[2] Dhananjay Yadav, R. Bhargava, G. S. Agrawal, Gyeong S. Hwang, Jinho Lee and M. C. Kim, “Magneto-convection in a rotating layer of nanofluid,” Asia-Pac. J. Chem. Eng., DOI: 10.1002/apj.1796.

[3] A.V. Kuznetsov and D.A. Nield, “Thermal Instability in a Porous Medium Layer Saturated by a Nanofluid: Brinkman Model,” Transp Porous Med, vol.81, pp.409-422, 2010.

[4] D. Yadav, R. Bhargava and G.S. Agrawal, “Boundary and internal heat source effects on the onset of Darcy-Brinkman convection in a porous layer saturated by nanofluid,” International Journal of Thermal Sciences, vol.60, pp. 244-254, 2012.

[5] Urvashi Gupta, Jyoti Ahuja and R.K. Wanchoo, “Magneto convection in a nanofluid layer,”International Journal of Heat and Mass Transfer, vol.64, pp.1163-1171, 2013.

[6] S.Chandrasekhar, Hydrodynamic and hydromagnetic stability, Oxford University Press, Oxford, 1961.