Nonlinear vibration of Carbon NanoTubes with piezoelectric layers conveying fluid under harmonic voltage

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Abstract

Based on the nonlocal elasticity theory and Euler-Bernoulli beam model, the nonlinear vibration of Carbon NanoTubes with a piezoelectric layer conveying fluid is investigated. The nonlocal effects in the piezoelectric material is also considered. The nonlinear partial differential equation governing the dynamic motion is presented. The applied voltage is considered to have both constant and time variable terms. The forced vibration under external load and parametric excitation is analyzed and different resonance conditions are studied based on the multiple scales method. The actuated CNTs are studied and the parametric analysis was elaborated in linear and nonlinear behaviors.

1. Introduction

In these last years, Carbon NanoTubes (CNTs) have been considered by many authors due to their excellent mechanical, thermal and electrical properties [1]. Length scale effect analysis on vibration behavior of single walled CarbonNanoTubes with generalized boundary conditions and CNT conveying fluid have been elaborated by Azrar et al [2-3]. The vibration of carbon nanotubes embedded in viscous elastic matrix under parametric excitation by nonlocal continuum theory was studied by Wang et al [4].
In this paper, the nonlinear partial differential equations modeling the dynamic behavior of CNT with piezoelectric layer conveying fluid and subjected to excitation force is presented. The length scale effects was introduced for the CNT materials as well as for the piezo part. Large amplitude displacement and time dependent voltage actuation are considered. The analytical solutions are obtained based on the multiple scales method. Primary and secondary resonances can be investigated and the voltage excitation effects based the piezoelectric layer can be analyzed for actuated CNT.

2. Mathematical formulation

Based on the nonlocal Euler-Bernoulli beam model with large displacements, the governing equation of viscoelastic CNT conveying viscous fluid and subjected to a transverse excitation is given by:

$$m_t u_{,tt} + EA\left(u_{,xx} - \left(u_{,x}^2\right)_{,x}\right) - \mu\, m_t u_{,xxtt} + A_1 e_1 \varphi_{,xx} + m_f\left(V^2 w_{,x} w_{,xx} + V w_{,x} w_{,xt}\right) = 0 \qquad (1,1)$$

$$\begin{aligned}
&EI\, w_{,xxxx} + c_s EI\, w_{,xxxxt} - m_f\left(V w_{,x} u_{,xt} + V w_{,xx} u_{,t} - 2V w_{,xt} + V w_{,x} w_{,xx} w_{,t} - V^2 w_{,xx}\right) + \left(m_f + m_t\right) w_{,tt} - EA\left(u_{,xx} + u_{,x} w_{,xx} + (3/2) w_{,xx} w_{,x}^2\right) - \vartheta A_f\left(V w_{,xxx} + w_{,xxt}\right) - A_1 e_1\left(\varphi_{,x} w_{,x}\right)_{,x} \\
&- \left(e_0 a\right)^2 \Big[\left(m_f + m_t\right) w_{,tt} - m_f\left(V w_{,x} u_{,xt} + V w_{,xx} u_{,t} - 2V w_{,xt} + V w_{,x} w_{,xx} w_{,t} - V^2 w_{,xx}\right) - EA\left(u_{,xx} + u_{,x} w_{,xx} + (3/2) w_{,xx} w_{,x}^2\right) \\
&- \vartheta A_f\left(V w_{,xxx} + w_{,xxt}\right) - A_1 e_1\left(\varphi_{,x} w_{,x}\right)_{,x}\Big]_{,xx} = F - \left(e_0 a\right)^2 F_{,xx} \qquad (1,2)
\end{aligned}$$

$$\frac{\partial \varphi_{,x}}{\partial x} = \frac{e_1}{k_1}\left[\frac{\partial}{\partial x}\left(u_{,x} + \frac{1}{2} w_{,x}^2\right)\right] = 0 \qquad (1,3)$$

where

$$EI = E_1 I_1 + E_2 I_2;\; EA = E_1 A_1 + E_2 A_2;\; \rho A = \rho_1 A_1 + \rho_2 A_2$$

in which w and u are the transverse and longitudinal displacements, respectively. φ is the electric potential, the constants, E_1, e_1 and k_1 indicate the Young modulus, piezoelectric and the dielectric constants of piezoelectric part. ρ_1 and ρ_2 are the density of piezoelectric layer and Carbon NanoTube respectively.
E_i, I_i, and A_i are the modulus of elasticity, cross section moment of inertia, cross section area of the i[th] layer.
V is the static mean flow velocity and $F(x,t)$ the transverse excitation force. c_s, $e_0 a$ and m_t are the viscoelastic coefficient of the tube, nonlocal parameter and the mass of the tube. m_f is the mass of the fluid per unit axial length, A_f and ϑ are the cross sectional area and the viscosity of the internal fluid respectively.
The Galerkin method is employed to solve the nonlinear differential equation (1). Based on this method, the following solution is considered for the transverse vibration of the system:

$$w(x,t) = \sum_{i=1}^{N} X_i(x) Y_i(t) \qquad (2)$$

Where X_i is the i[th] mode shape of vibration for the NanoTube, which could be obtained from the linear system and Y_i is the associated time responses. For the sake of clarity, the axial displacement effect is neglected in this analysis.
Substituting Eq. (2) into Eq. (1), and then multiplying both sides of the resulting equations with $X_j(x)$, and integrating it with respect to x over the domain (0, L), the governing equation of motion for one mode of nano beam vibration can be derived. After same mathematical developments and using the mode shape function associated to the considered boundary conditions.
The following dimensionless parameters are used:

$$y = \frac{x}{L};\; \hat{w} = \frac{w}{L};\; \hat{V}_0 = \frac{A_1 E_1 V_0 L}{EI};\; \hat{V}_1 = \frac{A_1 E_1 V_1 L}{EI};\; \hat{\vartheta} = \frac{\vartheta A_f}{\sqrt{EI m_f}};$$

$$t = \sqrt{\frac{(m_f + m_t)L^4}{EI}};\; M_r = \sqrt{\frac{m_f}{m_f + m_t}};\; \beta = \sqrt{\frac{EI}{m_f + m_t}}\,\frac{c_s}{L^2} \qquad (3)$$

$$\hat{U} = \sqrt{\frac{m_f}{EI}}\, vL;\; \mu_0 = \frac{e_0 a}{L};\; \hat{K} = \frac{pL^2}{EI};\; \hat{\xi} = \frac{EAL^2}{EI}$$

where V_0 and V_1 are the amplitudes of constant and harmonic voltage and $\hat{U}$ is the dimensionless fluid velocity.

The nonlinear differential equation for the time response of the fundamental mode is given by:

$$\ddot{Y}+\left(\omega_0^2+\varepsilon c_1\cos\left(\Omega_1 t\right)\right)Y+2\varepsilon c\dot{Y}+\varepsilon c_2 Y^3=\varepsilon F_0\cos\left(\Omega_2 t\right) \quad (4)$$

where ε is a small parameter and the constant coefficients c_1, ω_0, c_2 and F_0 are given by:

$$c_0=\int_0^1\Sigma_0 X_1 dy;\ \omega_0^2=\frac{1}{c_0}\int_0^1\Sigma_1 X_1 dy;\ c=\frac{1}{c_0}\int_0^1\Sigma_4 X_1 dy;$$

$$c_1=\frac{1}{c_0}\int_0^1\Sigma_2 X_1 dy;\ c_2=\frac{1}{c_0}\int_0^1\Sigma_3 X_1 dy; \quad (5)$$

$$F_0=\frac{\bar{F}_0}{c_0}\int_0^1\{\delta(x-x_0)+\mu_0^2\frac{\partial^2}{\partial x^2}\delta(x-x_0)\}X_1 dy;$$

In which $\bar{F}$ is the amplitude of applied point load.

$$\Sigma_0=X_1-\mu_0^2 X_1^{"}$$

$$\Sigma_1=X_1^{""}+\left(\hat{U}^2-\hat{V}_0+\hat{K}\right)X_1^{"}-\mu_0^2\left(\hat{U}^2-\hat{V}_0+\hat{K}\right)X_1^{""}-\vartheta\hat{U}X_1^{"} \quad (6)$$

$$\Sigma_2=-\hat{V}_1 X_1^{"}+\mu_0^2\hat{V}_1 X_1^{""};\ \ \Sigma_3=\hat{\zeta}\left(X_1^{"}(X_1^{'})^2+\mu_0^2\left(X_1^{"}(X_1^{'})^2\right)^{"}\right)$$

$$\Sigma_4=\beta X_1^{""}+2M_r\hat{U}X_1^{'}-\hat{\vartheta}X_1^{"}-2\mu_0^2 M_r\hat{U}X_1^{"}$$

With the multiple scale method [5], the approximate solution can be expressed in terms of different times cales as

$$Y(t)=Y_0(T_0,T_1)+\varepsilon Y_1(T_0,T_1)+... \quad (7)$$

Where the independent variable

$$T_n=\varepsilon^n t \quad (8)$$

$$\frac{d}{dt}=\frac{\partial}{\partial T_0}\frac{dT_0}{dt}+\frac{\partial}{\partial T_1}\frac{dT_1}{dt}+...=D_0+\varepsilon D_1+\varepsilon^2 D_2+... \quad (9)$$

$$\frac{d^2}{dt^2}=D_0^{\ 2}+2\varepsilon D_0 D_1+\varepsilon^2(D_2^{\ 2}+2D_0 D_2) \quad (10)$$

Substituting Eqs. (8-10) into Eq. (4), and separating the same order of the small dimensionless parameter ε, gives

$$\varepsilon^0: D_0^2 Y_0+\omega_0^2 Y_0=0 \quad (11)$$

$$\varepsilon^1: D_0^2 Y_1+\omega_0^2 Y_1=-2D_0 D_1 Y_0 - 2\mu D_0 Y_0 - c_1 D_1 Y_0$$
$$-c_2 Y_0^3+F_0\cos\left(\Omega_2 T_0\right) \quad (12)$$

The solution of Eq (11) is

$$Y_0(T_0,T_1)=A(T_1)e^{i\omega_0 T_0}+\bar{A}(T_1)e^{-i\omega_0 T_0} \quad (13)$$

Where $A(T_1)$ and $\bar{A}(T_1)$ are the unknown complex function and its complex conjugate form.

Substituting Eq. (13) into Eq. (12), one obtains:

$$D_0^2 Y_1+\omega_0^2 Y_1=-\left[2i\omega_0(A^{'}+\mu A)+3c_2 A^2\bar{A}\right]e^{i\omega_0 T_0}-\frac{c}{2}$$
$$\left[Ae^{i(\Omega_1+\omega_0)T_0}+\bar{A}e^{i(\Omega_1+\omega_0)T_0}\right]-c_2 A^3 e^{3i\omega_0 T_0}+\frac{F_0}{2}e^{i\Omega_2 T_0}+CC \quad (14)$$

where CC is the complex conjugate of the previous terms.

From the above equations, it is obvious that three resonance cases are studied

Case A: $\Omega_2=\omega_0+\varepsilon\sigma$

This case shows the primary resonance, the secular terms of Eq. (14) is

$$-2i\omega_0(A'(T_1)+\mu A(T_1))-3c_2 A(T_1)^2\bar{A}(T_1)+\frac{1}{2}F_0 e^{i\sigma T_1}=0 \quad (15)$$

The function $A(T_1)$ can be assumed as the polar form:

$$A(T_1)=\alpha(T_1)e^{i\theta(T_1)}/2 \quad (16)$$

where $\alpha(T_1)$ and $\theta(T_1)$ are the real functions

Substituting Eq. (16) into Eq. (15) and separating real and imaginary parts give:

$$\begin{cases}\frac{1}{2}(\gamma'-\sigma)\omega_0\alpha+\frac{3}{8}c_2\alpha^3+\frac{1}{4}c_1\alpha\cos(\gamma)=0\\ \omega_0\alpha'+c\omega_0\alpha+\frac{1}{4}c_1\alpha\sin(\gamma)=0\end{cases} \quad (17)$$

Based on the steady motion assumption, one should have $\alpha'=0$ and $\gamma'=0$. Thus

$$\begin{cases}-\frac{1}{2}\sigma\alpha+\frac{3}{8\omega_0}c_2\alpha^3=-\frac{1}{4\omega_0}c_1\alpha\cos(\gamma)\\ c\alpha=-\frac{1}{4\omega_0}c_1\alpha\sin(\gamma)\end{cases} \quad (18)$$

As a result, the frequency amplitude relationship can be obtained by simplifying Eq. (18) as

$$\sigma=\left\{\frac{3}{4}c_2\alpha^3\pm\sqrt{\left(\left(\frac{1}{4}c_1\alpha\right)^2-(c\omega_0\alpha)^2\right)}\right\}\Big/(\omega_0\alpha) \quad (19)$$

Case B $\Omega_1=2\omega_0+\varepsilon\sigma$

For this case, named parametric resonance, it is assumed that the frequency of parametric excitation caused by the applied voltage is near to the system natural frequency. Taking this condition, the elimination of secular terms from Eq. (14) gives

$$-2i\omega_0(A'(T_1)+\mu A(T_1))-3c_2 A(T_1)^2\bar{A}(T_1)+\frac{1}{2}c_1\bar{A}(T_1)e^{i\sigma T_1}=0 \quad (20)$$

Applying the polar form $A(T_1)=\alpha e^{i\theta}/2$ and taking out the real and imaginary parts

$$\begin{cases}\frac{1}{2}(\gamma'-\sigma)\omega_0\alpha+\frac{3}{8}c_2\alpha^3+\frac{1}{4}c_1\alpha\cos(\gamma)=0\\ \omega_0\alpha'+c\omega_0\alpha+\frac{1}{4}c_1\alpha\sin(\gamma)=0\end{cases} \quad (21)$$

where $\gamma=\sigma T_0-2\theta$ and taking $\gamma'=0$ and $\alpha'=0$ leads to

$$\begin{cases}-\frac{1}{2}\sigma\alpha+\frac{3}{8\omega_0}c_2\alpha^3=-\frac{1}{4\omega_0}c_1\alpha\cos(\gamma)\\ c\alpha=-\frac{1}{4\omega_0}c_1\alpha\sin(\gamma)\end{cases} \quad (22)$$

The frequency-amplitude relationship of the steady-state motion is obtained as

$$\sigma = \left\{ \frac{3}{4\omega_0} c_2 \alpha^2 \pm \sqrt{\left(\left(\frac{1}{4\omega_0} c_1 \right)^2 - c^2 \right)} \right\} \quad (23)$$

3. Numerical results and discussion

Numerical results are presented using effective properties of simply supported CarbonNanoTubes with a piezoelectric layer. The following geometrical and material properties are used.

$$E_1 = 0{,}8TPa, E_2 = 1TPa,\ \rho_c = 2.3e^3 kg/m^3, \rho_p = 1.3e^3 kg/m^3$$

$$\rho_w = 1e^3 kg/m^3, R = 3.5\,nm,\ h_c = 0.34\,nm,\ h_p = 0.034\,nm,\ c_s = 1e^{-3}$$

The results of primary resonance due to the external excitation have been presented in figures 1, 2 and 3.

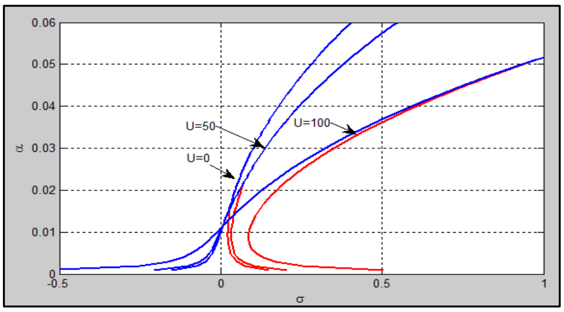

Figure 1: Fluid velocity U effect on frequency response of primary resonance

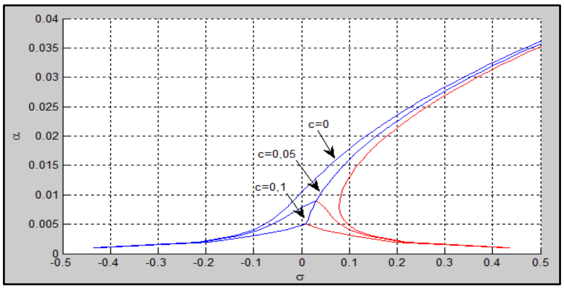

Figure 2: The viscoelastic coefficient c effect on frequency response of primary resonance

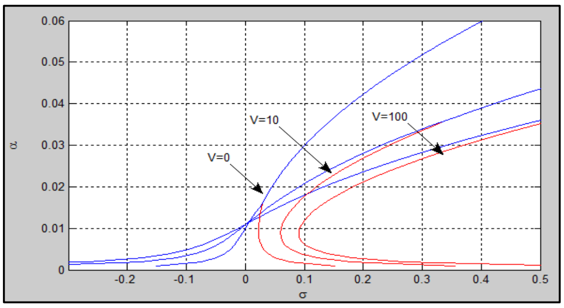

Figure 3: Effect of the voltage V on the frequency response of primary resonance

The effect of the fluid velocity on the frequency responses has been shown in Figure 1. The more fluid velocity, the higher amplitude of vibration and harder behavior would happen in the system vibration. The effects of the viscoelastic coefficient and applied harmonic voltage are presented in Figures 2 and 3, respectively. The frequency response is very sensitive to both parameters in terms of nonlinear hardening behavior and amplitude of vibration.

The vibration of the CNT under parametric excitation which is caused by the applied harmonic voltage is represented in figure 4. This figure shows the frequency-amplitude response of the system in which the red lines show the unstable solution. The jump phenomena may occur on the system when the voltage frequency approaches to the natural frequency.

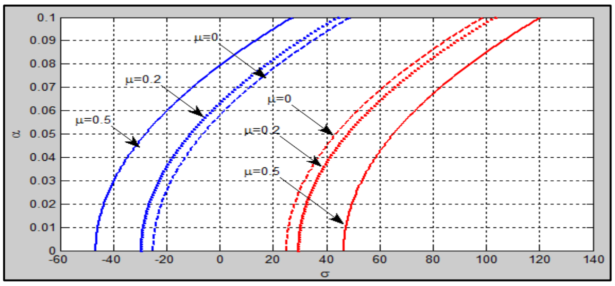

Figure 4: Nonlocal parameter μ effect on frequency response of the system for parametric resonance

Various other dynamic behaviors of actuated CNTs are investigated and a parametric analysis is elaborated.

4. Conclusion

The frequency and time responses are elaborated for different resonance cases. The effects of applied harmonic voltage, external excitation and the influence of piezoelectric effect, fluid motion and the small scale parameter are considered in the obtained results. The instability behaviors and frequency zone and studied related the fluid velocity and voltage frequency.

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