

# Simulation of functionally graded material beam with Piezoelectric Actuators

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## Abstract:

The aim of this work is to propose a prototype for functionally graded beam (FGbeam) with surface bounded piezoelectric actuator. In this regard, a finite element clamped-free FGbeam with piezoelectric actuation is modeled using ANSYS APDL language. The material properties of FG beam are assumed to be graded along the thickness direction and their materials properties are calculated according to power law distribution. The FGbeam is excited by piezoelectric actuator subjected to time harmonic voltages. The analysis is mainly for investigating the effect of the harmonic excitation using piezoelectric actuator on the functionally graded beam.

**Keywords:** *functionally graded material, piezoelectric, harmonic excitation*

## 1. Introduction

In recent years, the functionally graded materials have drawn much attention because of their significant thermal and mechanical properties. However since their discover in Japan in 1984, The FGMs are used in variety applications such as, aerospace, defense industry, automotive industries and machine elements. Because of the wide range of applications of FGMs, it's necessary to get a full understanding about their dynamic and static behavior. Many researchers focused on the study of static and dynamic analysis of FGM structures such as beams and plates (Wattanasakulpong and Ungbhakorn 2012; Şimşek 2010; Eltaher 2011; Reddy and Cheng 2001).

Due to their potential advantage in sensing and actuating, The piezoelectric materials are considered as the most appropriate and perfect solution of many technical issue such as vibration control ,shape control and noise reduction. However, Most of the research has been directed towards vibration control problems

(Fuller, Elliott, and Nelson 1996; Preumont 2011; Takács and Rohal'-Ilkiv 2012). A considerable research papers are interested in modeling of the FGM structures such as plates and beams with integrating piezoelectric (Dai et al. 2004; Reddy 2000; Yiqi and Yiming 2010; Bendine and Wankhade 2016)

In the present investigation, we developed a efficient ANSYS model for the case of FGbeam with bonded piezoelectric actuato. The harmonic excitation of FGbeam was provided by a piezoelectric actuator. The effect of the power low index on the beam response was investigated.

## 2. Constitutive equations

A functionally graded clamped free beam of length L, width b thickness h bounded by piezoelectric actuators is presented in fig. 1. The bottom surface of the FGM is a metal layer and the top surface is a ceramic layer. Different presentations were proposed to describe the variation of the FGM properties. In the present study the effective Young's moduls, Poisson's ratio and mass density are calculated according to power law distribution

$$\begin{cases} E_{fgm}(z) = (E_c - E_m)V_c + E_m \\ \rho_{fgm}(z) = (\rho_c - \rho_m)V_c + \rho_m \\ V_c = \left(\frac{2z + h}{2h}\right)^n \end{cases} \quad (1)$$

Where,  $E_{fgm}(z)$ ,  $\rho_{fgm}(z)$ , and  $R_n$  are respectively the Young's modulus, mass density and the power law index. The subscripts "m" and "c" refer to the metallic and ceramic constituents, respectively;  $V_c$  represent the volume fraction of the ceramic

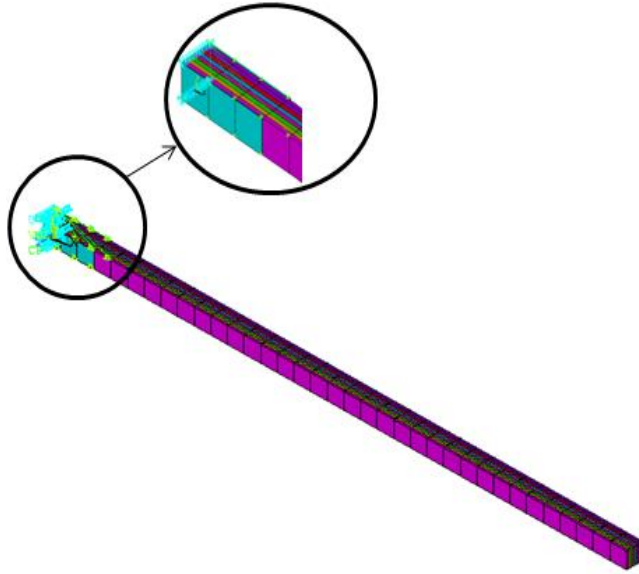


Fig 1. FGbeam equipped with piezoelectric patch with ANSYS.

The beam is equipped with piezoelectric patch, which is used as an actuators, the coupling relationship between the electrical and mechanical proprieties (Tzou and Tseng 1990) can be described by

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \\ D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{21} & 0 & 0 & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_{33} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} & 0 \\ 0 & 0 & 0 & 0 & e_{15} & 0 & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & -e_{31} \\ 0 & 0 & -e_{32} \\ 0 & 0 & -e_{33} \\ 0 & -e_{24} & 0 \\ -e_{15} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} \quad (2)$$

Where  $\sigma$  is the stress vector,  $D$  represent the electric displacement components,  $E$  denote the electric field

component,  $Q, e, \varepsilon$  are the elastic, the piezoelectric coupling and the dielectric permittivity constants respectively.

Using the previous equations with variational principle and applying element discretization, the dynamic equation FGbeam with piezoelectric patches can be expressed as

$$[M^e]\{\ddot{V}\} + [K^e]\{V\} = \{f_{ele}^e\} \quad (3)$$

where  $[M^e]$  and  $[K^e]$  are the element mass and stiffness matrices corresponding to the vector of mechanical displacement.  $\{f_{ele}^e\}$  is the electric forces applied by the actuator.

### 3. FEM simulation

Simulation was carried out for cantilever FGbeam with length 0.7 m, width 0.06 m, and height=Length/35, made of aluminum. The piezoelectric actuators of dimension 0.06 m  $\times$  0.02 m  $\times$  0.001 m. Geometry of the system was visualized by the finite element model and is shown in Fig. 1.

The FEA model of the FGbeam with the piezoelectric actuator was modeled in the ANSYS FEA software package (ANSYS 10.0 (2006)). Three dimensional structural element (SOLID186) is used for the FGM part of the smart beam. The piezoelectric patches are modeled using three dimensional coupled elements (SOLID5). The solid representation of the FGbeam and the actuator was created using the block command, then the Material properties have been assigned using a macro developed by the authors, finally the meshing and the boundary conditions were applied. Cantilever boundary conditions are defined for the nodes at  $x=0$ . The degrees of freedom, VOLT, are coupled for the nodes at the top and bottom surfaces of the actuator by the ANSYS command cp.

The flowchart developed in this study is presented in the fig 2.

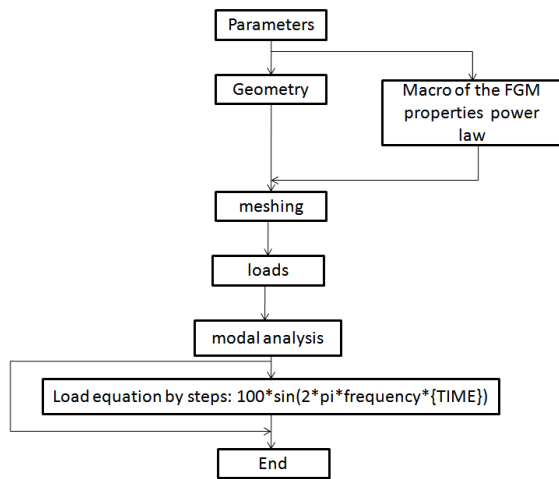


Fig 2. Flowchart of the Ansys Apdl Code used.

## 4. Results and discussion

### 4.1 Validation

In order To verify the effectiveness of the present FEM model, a clamped-free FGbeam was considered. the material proprieties were given in Table. 1. the natural frequencies of FGbeam are calculated using the non-dimensionalized expression given by equation 4 and compared with those obtained by (Şimşek 2010; Wattanasakulpong and Ungbhakorn 2012)

$$\lambda = \frac{\omega L b^2}{h} \sqrt{\frac{\rho_m}{E_m}} \quad (4)$$

The results are shown in Tables 2. It can be observed that the results are consistent and almost identical demonstrating the accuracy and validity of our FEM model.

Table. 1 Materials proprieties.

Proprieties	Aluminum (metal)	Zicronia (ceramic)	PZT G-1195
Elastic modulus E (N/m <sup>2</sup> )	70×10 <sup>9</sup>	151×10 <sup>9</sup>	6.1×10 <sup>10</sup>
Poisson's ratio	0.3	0.3	0.35

Density ρ (kg/m <sup>3</sup> )	2702	3000	7750
Elastic stiffness matrix (GPa)			61.0
E11			61.0
E22			53.2
E33			22.6
G12			21.1
G13			21.1
G23			
Piezoelectric strain matrix			
e31	-	-	6.5
e33			23.3
e15			17
Dielectric matrix (F/m)			
g11			1.53×10 <sup>-8</sup>
g22			1.53×10 <sup>-8</sup>
g33			1.5×10 <sup>-8</sup>

Table 2 Variation of fundamental frequency with the power-law exponent for C–F beam for L/h = 20.

Method used	Al <sub>2</sub> O <sub>3</sub>	n = 0.2	n = 0.5	n = 1	n = 2	n = 5	n = 10	Al
(Şimşek 2010)	1.9	1.8	1.6	1.5	1.3	1.3	1.2	1.0
(Wattanasakulpong and Ungbhakorn 2012)	524	171	626	029	714	057	671	129
akulpong and Ungbhakorn 2012)	1.9	1.8	1.6	1.5	1.3	1.3	-	1.0
ansys	53	16	63	04	72	07	1.2	15
ansys	1.9	1.7	1.6	1.5	1.4	1.3	780	1.1
ansys	38	799	344	114	158	314		807

### 4.2 Harmonic excitation

In this section, the effect of the harmonic excitation using piezoelectric actuator on the FGbeam is studied. An FGbeam (0.7 m × 0.06 × 0.7/35) bounded by piezoelectric actuator on the top and the bottom (0.06 m × 0.02 m × 0.001) is considered. The piezoelectric actuator is subjected to a persistent harmonic load  $q = 200 \times \sin(\omega \times t)$  V. The time step t is given by  $dt=1/20*f1$ , where f1 is the first natural frequency. Fig. 2 presented the harmonic voltage received by the actuator (Appendix).

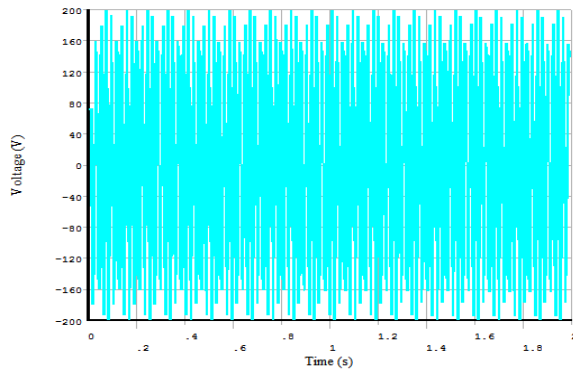
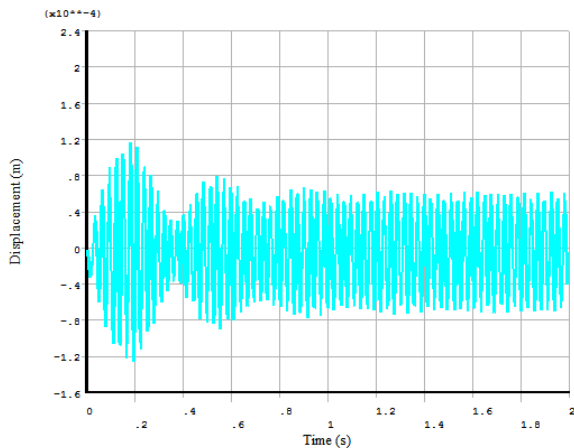
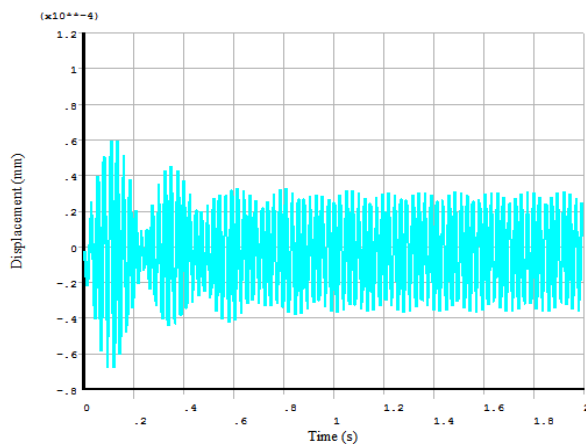
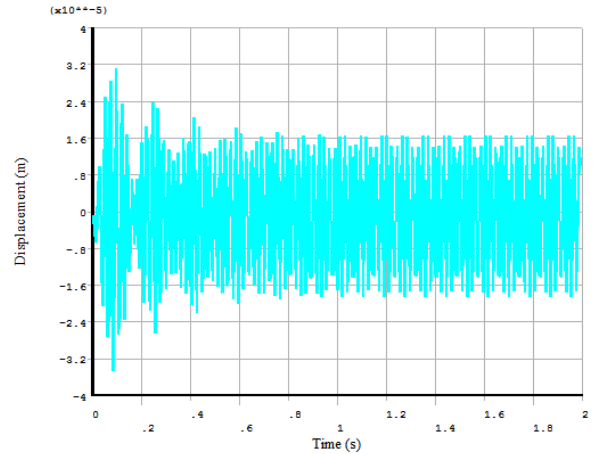


Fig.3 Actuator voltage.

The response of the FGbeam for the case of  $n=0.2$ , is given in fig 4. The maximum FBeam response is  $1.2 \times 10^{-4}$ . Fig.5. Shows the deflection for the case of  $n = 1$  with maximum deflection  $0.6 \times 10^{-4}$ , while the case of  $n=10$  is presented in fig 6 where the maximum deflection is  $3.2 \times 10^{-5}$

Fig.4 deflection of the FGM beam for the case of  $n= 0.2$ .Fig.5 deflection of the FGM beam for the case of  $n= 1$ .Fig.6 deflection of the FGM beam for the case of  $n= 10$ .

## 5. Conclusion

In the present investigation, a prototype of FGbeam with piezoelectric patch is proposed for the vibration study of FGbeam. The simulations treated the following points

- (1) The implementation of the FGbeam with piezoelectric patches on ANSYS.
- (2) The validation of the proposed model with studies found in literatures.
- (3) The effect of the power low index on the responses of the FGbeam under harmonic excitation.

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## Appendix

A part of the APDL code for the harmonic excitation provide by the actuator

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```

Time=2
Dt=Time/(20* f1)          !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
NTIMES= Time/ Dt         !!!!!
pi=3.141593              *dim,force,array,Ntimes
/SOLU                    force(1)=0.
ANTYPE,4                 *do,i,2,Ntimes,1
TRNOPT,FULL,,DAMP       force(i)=200*sin(2*pi*f1*sol
LUMPM,0                 time(i))

```

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```

OUTRES,ALL,ALL          *enddo
DELTIM,0.01            !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
Specifies the time step !!!!!
sizes                  *do,i,3,Ntimes,1
TINTP,,0.25,0.5,0.5    d,p1,VOLT,force(i),0
! Load equation by steps: d,p2,VOLT,-force(i),0
100*sin(frequency*{ Dt }) !F,1117,Fz,force(i)
TIME,0.01              *get,dz,node, 1117,u,z
KBC,1                  solve
                        !Ramped      TIME,soltime(i)
loading                *enddo
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! FINISH
!!!!!!!!
*dim,soltime,array,Ntime
s
soltime(1)=0.
*do,i,2,Ntimes,1
soltime(i)=soltime(i-1)+Dt
*enddo

```

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