Frequency and time closed form models for hybrid piezoelectric composites

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Abstract

In this paper, explicit models of time and frequency dependent effective properties of transversely isotropic viscoelectroelastic composites are elaborated. Based on the principle of linear viscoelectroelasticity and the Mori-Tanaka micromechanical mean field approach, the effective properties are derived in the Laplace-Carson domain and then inverted analytically or numerically to the time domain . Using compact matrix formulation, explicit relationships are obtained for different effective viscoelectroelastic moduli of piezoelectric materials with various inclusions shapes. Results based on the elaborated effective viscoelectroelastic models are presented for various types of piezoelectric inclusions and compared to the available predictions. These models are suitable for designers to predict viscoelectroelastic moduli of composite materials with optimized characteristics.

Mots clefs : *Explicit models; micromechanics; Mori-Tanaka; viscoelectroelastic; elliptic inclusions; frequency and time dependent properties; Laplace-Carson transform*

1. Introduction

Viscoelastic behavior is found in many materials such as polymers, wood, human tissue, etc. Some works have been elaborated to predict effective properties of such materials. Jiang and Batra [1] derived effective properties of viscoelectroelastic composites based on the Mori-Tanaka method. The viscoelectroelastic effective properties modeling of heterogeneous and multi-coated piezoelectric composite materials have been widely investigated by Azrar et al [2, 3]. Li and Dunn [4] obtained some closed form expressions of the effective moduli for viscoelectroelastic composites consisting of parallel PZT cylinder of elliptic cross section embedded in a viscoelastic matrix.

In this work, analytical and explicit expressions of effective properties of reinforced viscoelastic and viscoelectroelastic composite materials are elaborated. The coefficients of the matrix are considered to be time dependent. Elegant matrix formulation is used to get explicit analytical and semi analytical models for effective properties of transversely isotropic viscoelastic and viscoelectroelastic materials. Numerical results are obtained for various types of piezoelectric inclusions in the frequency and time domains.

2. Mathematical formulation

2.1 Basic equation of viscoelectroelasticity

Integral constitutive equations have been used to express the dependence of the viscoelectroelastic material response on the mechanical and electric field history. Based on the linear viscoelectroelastic behavior of piezoelectric composites, the constitutive equations, where the elastic and dielectricrelaxations are coupled, through the piezoelectric effect,can be written as:

$$\sigma_{ij}(x,t)=\left[\int_{-\infty}^{t}C_{ijkl}(x,t-\tau).\frac{\partial\varepsilon_{kl}(x,\tau)}{\partial\tau}-e_{ijk}(x,t-\tau).\frac{\partial E_k(x,\tau)}{\partial\tau}\right]d\tau, \quad (1)$$

$$D_i(x,t)=\left[\int_{-\infty}^{t}e_{ikl}(x,t-\tau).\frac{\partial\varepsilon_{kl}(x,\tau)}{\partial\tau}-\kappa_{ik}(x,t-\tau).\frac{\partial E_k(x,\tau)}{\partial\tau}\right]d\tau,$$

where σ_{ij} and ε_{ij} are stress and strain tensors, D_i and E_k are electric displacement and field, respectively. C_{ijkl}, e_{ijk}, κ_{ik} are relaxation elastic, piezoelectric and dielectric tensors, x is the position vector of a material point; t and τ are the current and reference time, respectively.

Using the condensed notations, equation (1) can be rewritten as.

$$\Sigma_{iJ}(x,t)=\int_{-\infty}^{t}E_{iJKl}(x,t-\tau).\frac{\partial Z_{Kl}(x,\tau)}{\partial\tau}d\tau, \quad (2)$$

where

$$\Sigma_{iJ}(x,t)=\begin{cases}\sigma_{iJ}(x,t) & J=1,2,3\\ D_i(x,t) & J=4\end{cases}$$

$$Z_{Kl}(x,t)=\begin{cases}\varepsilon_{Kl}(x,t) & K=1,2,3\\ -E_l(x,t) & K=4\end{cases} \quad (3)$$

$$E_{iJKl}(x,t)=\begin{cases}C_{iJKl}(x,t) & J,K=1,2,3\\ e_{iJl}(x,t) & J=1,2,3;K=4\\ e_{iKl}(x,t) & J=4;K=1,2,3\\ \kappa_{il}(x,t) & J=4;K=4\end{cases} \quad (4)$$

In the absence of body force and free charge, the equilibrium equation is written as

$$\Sigma_{iJ,J}(x,t)=0; \quad (5)$$

Converting equation (2) in the frequency domain yields

$$\hat{\Sigma}_{iJ}(s) = s\hat{E}_{iJKl}(s)\hat{Z}_{kl}(s) \quad (6)$$

Based on the mathematical developments, elaborated by Azrar et al. [3], the effective electroelastic moduli of the composite in the Laplace domain is defined by

$$\hat{E}^{-1}\hat{E}^{m} = \hat{I}_d - f\hat{T}^{-1}$$
$$\hat{T} = (1-f)\hat{S} + f\hat{I}_d + \left(\hat{E}^{p} - \hat{E}^{m}\right)^{-1}.\hat{E}^{m} \quad (7)$$

where $\hat{E}$, $\hat{E}_m$, $\hat{E}_p$ and S are the matrix notations of the electroelastic moduli of homogeneous composite, matrix and inclusion and Eshelby tensor.

2.2 Bloc matrix formulation

Matrices $\hat{E}$, $\hat{E}_m$, $\hat{E}_p$ and S for transversely isotropic material can be expressed in the form:

$$X = \begin{bmatrix} A & 0_{3x5} & P^{t} \\ 0_{5x3} & B & 0_{5x1} \\ P & 0_{1x5} & C \end{bmatrix} \quad (8)$$

where A, B, C and P are 3x3, 5x5, 1x1 and 3x1 matrices. Some mathematical developments lead to the following decoupled main relationships:

$$\hat{B}^{-1}\hat{B}^{m} = \hat{I}_5 - f(\hat{B}^{T})^{-1} \quad \text{(9-a)}$$

$$\begin{bmatrix} \hat{A} & \hat{P}^{t} \\ \hat{P} & \hat{C} \end{bmatrix}^{-1} \begin{bmatrix} \hat{A}^{m} & \hat{P}^{m} \\ \hat{P}^{m} & \hat{C}^{m} \end{bmatrix} = \begin{bmatrix} \hat{I}_3 & \hat{0} \\ \hat{0} & \hat{I}_1 \end{bmatrix} - f \begin{bmatrix} \hat{T}_3^{3} & \hat{T}_1^{3} \\ \hat{T}_3^{1} & \hat{T}_1^{1} \end{bmatrix}^{-1} \quad \text{(9-b)}$$

3. Viscoelastic models of piezocomposite

A composite made of transversely isotropic piezoelectric fibers embedded in isotropic viscoelastic matrix is considered. To model the viscoelastic behavior of the matrix, the Zener model is used. The relaxation functions for the considered model in the time and in Carson domains are given by

$$\hat{R} = E\left(1 + \frac{\rho s}{s+\tau}\right) \quad (10)$$
$$R = E(1 + \rho e^{-\tau t})$$

where the coefficients E, ρ and τ are experimentally given for the considered viscoelastic material.

3.1Effective decoupledmoduli

Following some mathematical developments [3], the viscoelastic behavior of decoupled moduli of matrix in time domain can be expressed as

$$\mu_{23}^{m} = \mu^{m}(1 + \rho_c e^{-\tau_e t});$$
$$k_{22}^{m} = k^{m}(1 + \rho_k e^{-\tau_c t}); \quad (11)$$
$$e_{15}^{m} = 0;$$

where μ^m, k^m, ρ_c *and* ρ_k are time-independent material constants of the matrix, and $\tau_e > 0$ and $\tau_c > 0$ are respectively, the elastic and the dielectric relaxation times of the matrix.

The electroelastic moduli of the inclusion are assumed to be time independent and given by

$$\mu_{23}^{p} = \mu^{p}; k_{22}^{p} = k^{p}; e_{15}^{p} = e^{p}; \quad (12)$$

For the considered composites, using Eqs (9-a), the resulted uncoupled effective electroelastic moduli in the Laplace-Carson domain are given by:

$$\hat{\mu}_{23} = \frac{\alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0}{\beta_3 s^3 + \beta_2 s^2 + \beta_1 s + \beta_0}$$
$$\hat{k}_{22} = \frac{\delta_3 s^3 + \delta_2 s^2 + \delta_1 s + \delta_0}{\gamma_3 s^3 + \gamma_2 s^2 + \gamma_1 s + \gamma_0} \quad (13)$$
$$\hat{e}_{15} = \frac{\lambda_2 s^2 + \lambda_1 s + \lambda_0}{\psi_2 s^2 + \psi_1 s + \psi_0}$$

where the coefficients $\boldsymbol{\alpha_k}$,$\boldsymbol{\beta_k}$,$\boldsymbol{\delta_k}$, γ_k, $\boldsymbol{\lambda_k}$ and $\boldsymbol{\psi_k}$ are functions of μ^m, k^m,ρ_c, ρ_k, τ_e and τ_c as well as of the electroelastic moduli inclusion [3].

The time dependent moduli are obtained based on the analytical Laplace-Carson inversion. Closed form time dependent μ_{23}, k_{22} and e_{15} are obtained and given by: (14)

$$\mu_{23}(t) = \frac{1}{\beta_0}\left[\alpha_0 + \sum_{\eta_k} \frac{e^{-\eta_k t}(-\beta_1\alpha_0 + \beta_0\alpha_1}{3\eta_k^2\beta_3 + 2\eta_k\beta_2 + \beta_1} + \sum_{\eta_k} \frac{e^{-\eta_k t}((\beta_0\alpha_3 - \beta_3\alpha_0)\eta_k^2 + (\beta_0\alpha_2 - \beta_2\alpha_0)\eta_k}{3\eta_k^2\beta_3 + 2\eta_k\beta_2 + \beta_1}\right]$$

$$k_{22}(t) = \frac{1}{\gamma_0}\left[\delta_0 + \sum_{\eta_k} \frac{e^{-\eta_k t}(-\gamma_1\delta_0 + \gamma_0\delta_1}{3\eta_k^2\gamma_3 + 2\eta_k\gamma_2 + \gamma_1} + \sum_{\eta_k} \frac{e^{-\eta_k t}((\gamma_0\delta_3 - \gamma_3\delta_0)\eta_k^2 + (\gamma_0\delta_2 - \gamma_2\delta_0)\eta_k}{3\eta_k^2\gamma_3 + 2\eta_k\gamma_2 + \gamma_1}\right]$$

$$e_{15}(t) = \frac{\lambda_0}{\psi_0} + \frac{(\lambda_2\psi_0 - \lambda_0\psi_2)\cosh\left(\frac{1}{2}\frac{t\sqrt{\psi_1^2 - 4\psi_2\psi_0}}{\psi_2}\right)e^{-\frac{1}{2}\frac{\psi_1}{\psi_2}t}}{\psi_0\psi_2} + \frac{(2\lambda_1\psi_0\psi_2 - \lambda_2\psi_0\psi_1 - \lambda_0\psi_1\psi_2)\sinh\left(\frac{1}{2}\frac{t\sqrt{\psi_1^2 - 4\psi_2\psi_0}}{\psi_2}\right)e^{-\frac{1}{2}\frac{\psi_1}{\psi_2}t}(\lambda_2\psi_0 - \lambda_0\psi_2)}{\psi_2\psi_0\sqrt{\psi_1^2 - 4\psi_2\psi_0}}$$

3.2 Effective coupled moduli

The effective viscoelectroelastic moduli of a piezoelectric composite with parallel cylindrical PZT inclusions in a dielectrically relaxing viscoelastic matrix are derived. The Poisson's ratio, ν, for the matrix is assumed to be real and constant. Closed form expressions are obtained based on the Laplace-Carson transforms and the Mori-Tanaka approach. The elastic, piezoelectric and dielectric moduli are given by the following equations. For simplification, the Hill moduli are used:

$$\hat{\Gamma} = \frac{\hat{\Gamma}^m \hat{\Gamma}^p + f\hat{\Gamma}^p \hat{\Delta}^m + (1-f)\hat{\Gamma}^m \hat{\Delta}^m}{(1-f)\hat{\Gamma}^p + f\hat{\Gamma}^m + \hat{\Delta}^m}$$

$$\hat{\Delta} = \hat{\Delta}^m \frac{(1+f)\hat{\Gamma}^m \hat{\Delta}^p + 2\hat{\Delta}^m \hat{\Delta}^p + (1-f)\hat{\Gamma}^m \hat{\Delta}^m}{(1-f)\hat{\Gamma}^m \hat{\Delta}^p + 2(1-f)\hat{\Delta}^m \hat{\Delta}^p + (1+f)\hat{\Gamma}^m \hat{\Delta}^m + 2f(\hat{\Delta}^m)^2} \quad (15)$$

$$\hat{C}_{11} = \hat{\Gamma} + \hat{\Delta}, \qquad \hat{C}_{12} = \hat{\Gamma} - \hat{\Delta},$$

$$\hat{C}_{13} = \hat{C}_{13}^m + \frac{f\hat{C}_{11}^m(\hat{C}_{13}^p - \hat{C}_{13}^m)}{(1-f)\hat{\Gamma}^p + f\hat{\Gamma}^m + \hat{\Delta}^m} \quad (16)$$

$$\hat{C}_{33} = (1-f)\hat{C}_{33}^m + f\hat{C}_{33}^p - f(1-f)\frac{(\hat{C}_{13}^p - \hat{C}_{13}^m)^2}{(1-f)\hat{\Gamma}^p + f\hat{\Gamma}^m + \hat{\Delta}^m}$$

$$\hat{e}_{31} = \frac{f\hat{e}_{31}^p(\hat{\Gamma}^m + \hat{\Delta}^m)}{(1-f)\hat{\Gamma}^p + f\hat{\Gamma}^m + \hat{\Delta}^m} \quad (17)$$

$$\hat{e}_{33} = f(\hat{e}_{33}^p - \frac{(1-f)\hat{e}_{31}^p(\hat{C}_{13}^p - \hat{C}_{13}^m)}{(1-f)\hat{\Gamma}^p + f\hat{\Gamma}^m + \hat{\Delta}^m})$$

$$\hat{k}_{33} = (1-f)\hat{k}_{33}^m + f\hat{k}_{33}^p + \frac{(1-f)\, f\hat{e}_{31}^p}{(1-f)\hat{\Gamma}^p + f\hat{\Gamma}^m + \hat{\Delta}^m} \quad (18)$$

These explicit models can be used by the designer for the conception of passive-active composite materials with optimized characteristics

4. Numerical results

Using the elaborated models, effective properties of piezoelectric composites are presented. Composite materials are considered in which the first phase, matrix, is constituted of isotropic viscoelastic polymer LaRC-SI and the second phase,aligned inhomogeneities, is a transversely isotropic piezoelectric material (PZT-7A).The properties of the constituents at room temperature are given in Table 1. The longitudinal fibers are placed in the x_3-direction. Numerical predictions are obtained for various types of piezoelectric inclusions.

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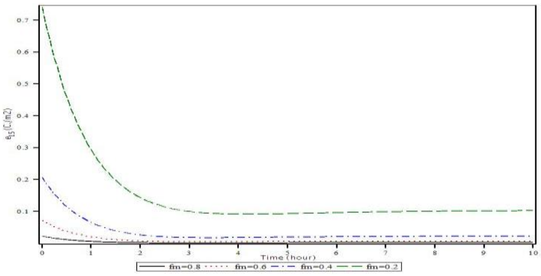
	C_{11}	C_{12}	C_{13}	C_{33}	C_{44}	e_{31}	e_{33}	e_{15}	$\frac{\kappa_{11}}{\kappa_0}$	$\frac{\kappa_{11}}{\kappa_0}$
PZT-7A	**148**	**76.2**	**74.2**	**131**	**25.4**	**-2.1**	**9.5**	**9.2**	**460**	**235**
LaRC-SI	**8.1**	**5.4**	**5.4**	**8.1**	**1.4**	**0**	**0**	**0**	**2.8**	**2.8**

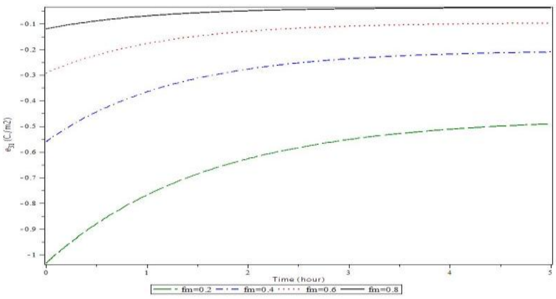
Table 1: Used electromechanical material properties

Based on the developed mathematical models and the SLS viscoelastic scheme, numerical predictions in time domain are presented. Effective coefficients are analyzed using analytical equations (14) to (18) and these coefficients are obtained for fibrous visco-electro-elastic composite. The values of ρ_c, ρ_k, τ_e and $\tau_c$are time-independent material constants of the matrix of equation (11), and have the following values $\rho_c=\rho_k=2.5$, $\tau_e=\tau_c=1$/hour.

The time dependent effective piezoelectric moduli e_{15} *(t)* and e_{31} *(t)*, are shown in figures 1 and 2. The effect of the variation of the volume fraction is presented on the effective properties. It is seen that e_{15} *(t)* decreases while e_{33} *(t)* increases with time. Various other predicted coefficients can be analysed in time and frequency domains.



(a)



(b)

Figure: Effective piezoelectric relaxation moduli (a) $e_{15}(t)$ and (b) $e_{31}(t)$ with respect to time for a fibrous viscoelectroelastic composite consisting of fibrous piezoelectric inclusions embedded in a viscoelastic matrix.

5. Conclusion

In this article, explicit analytical and semi-analytical models have been developed based on visco-electro-elastic model and matrix formulation for coupled and decoupled moduli for composite piezoelectric. These models are suitable for designing material with optimized properties for specified type of inclusions.

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