Robust and stable implicit time scheme for the Nonlinear Dynamics analysis of beam like structures.

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Abstract:

In this work we present an implicit time integration scheme for nonlinear dynamics of shear-deformable geometrically exact planar beam. The large difference in bending versus shear or axial stiffness leads to a set of stiff differential equations. This scheme is based on mid-point rule by enforcing the conserving and decaying energy to ensure the stability and robustness. Thedissipation in the higher mode is controllable by desirable properties. Several numerical simulations are presentedto illustrate the performance and overall stability and robustness of the proposed schemes.

Keywords : *dynamic, geometrically exact beam, conserving energy, decay energy..*

1. Introduction

In this work we propose an extension of an implicit energy conserving scheme for the dynamic analysis of 2D beam-like structures developed by Gams and al [1] introducing dissipation effects of interest for stiff equations. However, we do not add viscous dissipation but rather follow the concept first proposed by Ibrahimbegovic and Mamouri [2] by controlling dissipation only for high frequency modes. This consists of two energy dissipation mechanisms, the first controllingthe magnitudes of the strain energy and the second the kinetic energy of high frequency modes. Thus, for every time step in which high frequency mode content becomes significant we can reduce the total energy by dissipating their contribution enhancing the stability and robustness of the scheme. In particular, we are able to damp more efficiently the axial, shear and bending force contributions by high frequency modes compared to the scheme proposed by Gams and al [3].

2.Dynamics of 2D geometrically exact beam

We describe in this section the equations governing nonlinear dynamics of 2D beam undergoing large rotations and displacements, this model is based on the first theory proposed by Reissner, extended to handle an arbitrary space curved beam.

The initial configuration of the beam is specified by position φ_0 of the point,$s \in[0, L]$ (where L is the beam length), and the orientation of the cross-section, specified with the unit vector $\boldsymbol{t}_{1,0}$ which is normal to section. It is chosenasthe tangent to the beam axis, it can be written as:

$$\mathbf{t}_{1,0} = \boldsymbol{\varphi}_0'(s) \qquad (1)$$

Where $(\bullet)' = \partial(\bullet)/\partial s$ .According to Reissner's kinematic hypothesis (see [4]), the beam configuration is defined as:

$$\phi(s,t,\zeta) = \varphi(s,t) + \zeta\, \boldsymbol{t}_2(s,t) \qquad (2)$$

We note that t_1 and t_2 areobtained directly using the 2D rotation matrix

$$\begin{cases} \boldsymbol{t}_1 = \boldsymbol{\Lambda}\boldsymbol{t}_{1,0} \\ \boldsymbol{t}_2 = \boldsymbol{\Lambda}\boldsymbol{t}_{2,0} \end{cases}, \quad \boldsymbol{\Lambda} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \qquad (3)$$

Following the kinematic description, the strain measure for the 2D beam is expressed in the initial configuration by the following relationship:

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}^{\mathrm{T}}(\varphi' - \boldsymbol{t}_1) \quad \& \quad K = \psi' \qquad (4)$$

Where $\boldsymbol{\Sigma}$ contains axial and shear strains and K is the bending strain. The time derivative of material strain is computed us:

$$\dot{\boldsymbol{\Sigma}} = \dot{\boldsymbol{\Lambda}}^{\mathrm{T}}(\dot{\varphi}' - \mathbf{W}\varphi'\dot{\psi}) \qquad (6)$$

We denote $\mathbf{N}$ and M the stress resultants and couple, which are energy conjugate to $\boldsymbol{\Sigma}$ and K respectively. If we consider the linear elastic, the material stress and couple are defined by:

$$\begin{aligned} \boldsymbol{N} &= \boldsymbol{C\Sigma} \quad , \quad C = diag(EA, GA) \\ M &= C_m K \quad , \quad C_m = EI \end{aligned} \qquad (7)$$

Where E is the Young modulus, G is the shear modulus, A is the beam section, and I is the inertia of the section. The strain measures and stress resultants should satisfy the weak form of the equations of motion of the beam, which is written as described in [4] :

$$\int_0^L (\delta\varphi \cdot A_\rho \ddot{\varphi} + \delta\psi J_\rho \ddot{\psi}) ds + \int_0^L (\delta\varepsilon \cdot \boldsymbol{n} + \delta k\, m) ds - \pi_{ext} = 0 \qquad (8)$$

The virtual strain $\delta\varepsilon$ is given by (see [2,4,5])

$$\delta\varepsilon = \boldsymbol{\Lambda}\delta\boldsymbol{\Lambda}^T(\varepsilon) = \delta\varphi' - \boldsymbol{W}\varphi'\delta\psi \qquad (9)$$

3. Mid-point scheme:

The time discretization of the weak form of the equations of motion is established, using the mid-point rule approximation. Therefore, the equation (8) must be written at time $t_{n+1/2} = t_n + \Delta t/2$ is the mid-point of the time step increment $\Delta t = t_{n+1} - t_n$) :

$$\int_0^L \delta\varphi \cdot A_\rho \ddot{\varphi}_{n+\frac{1}{2}} ds + \delta\psi J_\rho \ddot{\psi}_{n+\frac{1}{2}} ds + \int_0^L (\delta\varphi' - \boldsymbol{W}\varphi'_{n+\frac{1}{2}}\delta\psi) \cdot \boldsymbol{n}_{n+\frac{1}{2}} ds + \delta\psi' m_{n+\frac{1}{2}} ds = \pi_{ext,n+\frac{1}{2}} \qquad (10)$$

where $\ddot{\varphi}_{n+\frac{1}{2}}$ and $\ddot{\psi}_{n+\frac{1}{2}}$ are computed using mid-pointapproximation rules as:

$$\ddot{\varphi}_{n+\frac{1}{2}} = \frac{\dot{\varphi}_{n+1} - \dot{\varphi}_n}{\Delta t} \qquad (11)$$

$$\ddot{\psi}_{n+\frac{1}{2}} = \frac{\dot{\psi}_{n+1} - \dot{\psi}_n}{\Delta t} \qquad (12)$$

3-1 Enforcement of theenergy conservation:

This scheme is designed to preserve the total energy. We can further, Compared to the scheme proposed by Ibrahimbegovic and Mamouri [4] the angular velocity is computed using linear update, similarly as translation velocities, as:

$$\Delta t\omega_{n+\frac{1}{2}} = \Delta\psi \Rightarrow \omega_{n+1} = -\omega_n + 2\frac{\Delta\psi}{\Delta t} \qquad (14)$$

In order to ensure that this scheme conserve the energy in the absence of external forces, the stress resultants at $t_{n+1/2}$ must be computed with the following expression:

$$\boldsymbol{N}_{n+\frac{1}{2}} = \boldsymbol{C}\frac{1}{2}(\Sigma_{n+1} + \Sigma_n) \qquad (15)$$

$$M_{n+\frac{1}{2}} = C_m \frac{1}{2}(K_{n+1} + K_n) \qquad (16)$$

To get more details see Mamouri & al. [5].

3-2-Energy decaying scheme

This scheme is used in this work to introduce desirable properties ofcontrollable energy decaying to overcome the loss of accuracy especially for the internal forces in presence of the high frequency.More precisely the constitutive equations and update of velocities are constructed, in manner that the energy will be dissipated by filtering out the high frequency contribution over each timewhen is needed, using two control parameters for internal and kinetic energy (see Mamouri & al [5]).

First, we modify the algorithmic constitutive equations as to get the dissipation in the internal energy:

$$\boldsymbol{N}_{n+\frac{1}{2}} = \boldsymbol{C}\frac{1}{2}(\Sigma_{n+1} + \Sigma_n) + \alpha\boldsymbol{C}(\Sigma_{n+1} - \Sigma_n) \qquad (17)$$

$$M_{n+\frac{1}{2}} = C_m \frac{1}{2}(K_{n+1} + K_n) + \alpha C_m (K_{n+1} - K_n) \qquad (18)$$

Where α is the parameter for controlling the dissipation of the strain energy.

Second, to assure the decay of the total energy in the case of vanishing strains, we ought to introduce dissipation in the inertia term introducing an appropriate velocity modification. Namely,the upadate of velocities Must be done using the corresponding dissipation term as

$$\Delta\mathbf{u} = \frac{\Delta t}{2}(\dot{\boldsymbol{\varphi}}_{n+1} + \dot{\boldsymbol{\varphi}}_n) + \beta\Delta t(\dot{\boldsymbol{\varphi}}_{n+1} - \dot{\boldsymbol{\varphi}}_n) \qquad (19)$$

$$\Delta\theta = \frac{\Delta t}{2}(\omega_{n+1} + \omega_n) + \beta\Delta t(\omega_{n+1} - \omega_n) \qquad (20)$$

Where β is the parameter that controls the dissipation of the kinetic energy.

To get more details see Mamouri & al. [5].

4. Numerical simulations

4.1 The swinging pendulum

This numerical example is used to demonstrate the conservation properties of the scheme with emphasis on the linear and angular moment.

In this example we represents the geometrically-nonlinear analysis of a planar mechanism which consists of four bars Fig.1.

Shear strains are made negligible by setting a large value for the shear modulus (G = 100E). The rigid links are assumed weightless $\rho = 0 kg/m^3$ and their rigidity is modelled by assuming large Young's modulus, i.e. 10 times the value of the modulus of the flexible beam

Parameters:

Lumped mass m = 0.5 kg,mass density $\rho = 2700 kg/m^3$

Cross section area $A = 0.005 \times 0.01 m^2$,

Young's modulus $E = 73 \times 10^9 N/m^2$. Shear strains are made negligible by setting a large value for the shear modulus (G = 100E). The rigid links are assumed weightless and their rigidity is modelled by assuming large Young's modulus, i.e. 10 times the value of the modulus of the flexible beam

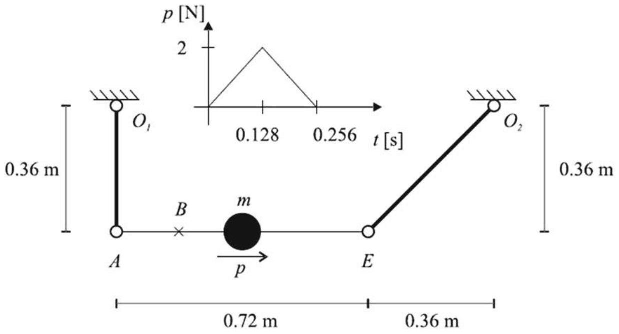

Fig. 1 Mechanical and geometriccharacteristics

Figure **2** shows the numerical results for time variation of the horizontal displacement computed by two schemes are very close and cannot practically be distinguished in the plot; considering a fairly smooth variation of the displacement the vertical confirms that the added dissipation will mainly affect the higher modes

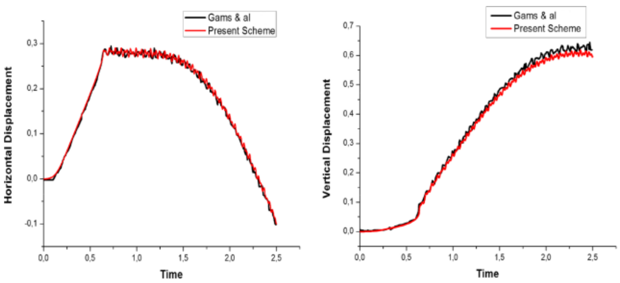

Fig. 2 Displacements of lumped mass with decaying energy scheme

This kind of conclusion can also be drawn for computed values of the shear force, where the energy conserving is not capable of damping out the high-frequency content, leading to the numerical results for the axial, shear and bending forces which are practically dominated by the numerical noise (see Fig. 3).

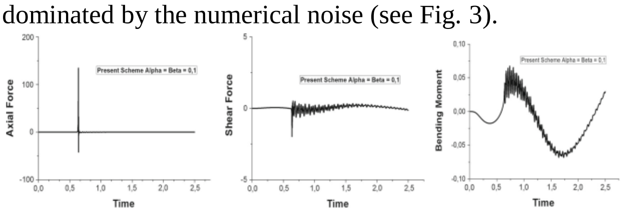

Fig. 3 Axial, shear forces and bending moment obtained with present scheme

When our results are compared with those of Gamset al. [3] (see Figs. **4**), we see that the quality of the filtering out of the noise in the forces is veryinteresting as is mentioned in this example.

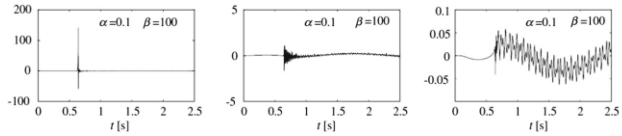

Fig. 6 Axial, shear forces and bending moment obtained by Gams et al. [3]

5. Conclusions

A novel design of time integration scheme has been presented in this work for nonlinear dynamics 2D geometrically exact beam, which are able to enforce the energy conservation or decay. This is needed for long-term simulations and especially for the case when the motion is described by the stiff equations due to large differences in stiffness associates with various deformation modes.

We have demonstrated inseveral numerical examples that the proposed dissipative scheme is needed, especially for getting improved values of beam stress-resultant internal forces. The proposed scheme leads to better damping of high-frequency content for axial, shear and bending forces compared to a recent approach proposed by Gams et al. [3].

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