Vibration and optimization based damage identification of plates with a variably oriented surface crack

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Abstract

The main aims of this work are on one hand to investigate free and forced vibrations of plates containing a surface inclined crack, and on the other hand to develop crack identification strategies by using optimization techniques.

The 2D differential quadrature method is elaborated for free vibration problems, and the genetic algorithm is adopted for damage identification. Eigenfrequencies and eigenmodes are obtained by few discretization points, and used for cracks parameters identification. The impact of crack orientation, length and depth is investigated for plates with different boundary conditions. Identification of damage severity is also discussed; and an efficient crack characterization is elaborated.

Keywords: *Plate; oriented crack; DQM; optimization; crack identification*

1. Introduction

Vibration based structural health monitoring is the process of identifying the health of structures based on changes in dynamic behavior caused by damage [1]. This behavior was described by many models, for 1D solids, i.e. beams containing one ore multiple open cracks [2, 3], or breathing ones. Vibration of damaged 2D solids such as plates and shells was investigated in case of horizontal crack [4,5,6]. As the main objective of this work is to develop strategies for cracks identification in different structures, we will first present the previously investigated model of cracked plates with inclined crack, and elaborate the mathematical and numerical developments of the differential quadrature method to this problem. An adjusted optimization procedure based on genetic algorithms is proposed to predict orientation, length and depth of cracks in plates.

2. Vibration of plates with variably oriented surface crack

A rectangular cracked plate of dimensions Lx, Ly and H and material properties of E, ρ, υ, is considered. The plate is assumed to contain a crack at the centre of its top surface, inclined with an angle β with respect to x axis, with as depth h, and length of 2C (see Fig.1).

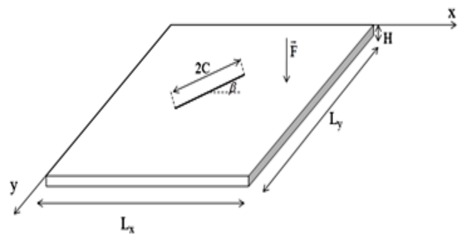

Figure 1: Rectangular plate with inclined surface crack under transversal vibration

The present problem is modeled through the following partial differential equation

$$\begin{aligned} D\nabla^4 w = \rho H\omega^2 w(x,y) &+ \frac{CD(1+\cos(2\beta))}{((\alpha_{bt}/6)+\alpha_{bb})(3+\upsilon)(1-\upsilon)h+2C}\frac{\partial^4 w}{\partial y^4} \\ &+ \frac{CD\upsilon(1+\cos(2\beta))}{((\alpha_{bt}/6)+\alpha_{bb})(3+\upsilon)(1-\upsilon)h+2C}\frac{\partial^4 w}{\partial x^2\partial y^2} \\ &+ \frac{2CD\sin(2\beta)}{((C_{bt}/6)+C_{bb})(1+\upsilon)h+2C}\frac{\partial^4 w}{\partial x\partial y^3} \\ &+ \frac{2CD\upsilon\sin(2\beta)}{((C_{bt}/6)+C_{bb})(1+\upsilon)h+2C}\frac{\partial^4 w}{\partial x^3\partial y} \end{aligned} \quad (1)$$

where $w = w(x, y, t)$ is the transverse displacement response, ∇^4 is the bi-harmonic operator and $D = \frac{Eh^3}{12(1-\upsilon^2)}$ is the plate's flexural rigidity. α_{bb} is the nondimensional bending compliance, $\alpha_{bt} = \alpha_{tb}$ is the nondimensional bending-stretching compliances, given by [5]

$$\alpha_{bb} = \frac{1}{H}\int_0^h g_b^2 dz \quad ; \quad \alpha_{bt} = \frac{1}{H}\int_0^h g_b g_t dz \quad (2)$$

in which g_b and g_t are dimensionless functions of the crack depth to the thickness ratio given by [5]:

$$g_b = (\frac{z}{H})^{1/2}\left(1.99 - 2.47(\frac{z}{H}) + 12.97(\frac{z}{H})^2 - 23.117(\frac{z}{H})^3 + 24.80(\frac{z}{H})^4\right) \quad (3\text{-a})$$

$$g_t = (\frac{z}{H})^{1/2}\left(1.99 - 0.41(\frac{z}{H}) + 18.7(\frac{z}{H})^2 - 38.48(\frac{z}{H})^3 + 53.85(\frac{z}{H})^4\right) \quad (3\text{-b})$$

The boundary conditions that must be satisfied by an edge parallel to **x** edge for example are as follows:

$$\text{Clamped edge: } w = 0 \;\; ; \;\; \frac{\partial w}{\partial y} = 0 \quad (4\text{-a})$$

Pinned edge: $w=0$; $M_y=-D(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2})=0$ (4-b)

Free edge: $M_y=0$; $\left(\frac{\partial^3 w}{\partial y^3}+(2-\nu)\frac{\partial^3 w}{\partial y \partial x^2}\right)=0$ (4-c)

3. Vibration of cracked plates based on the DQM

Thanks to the efficiency of the differential quadrature method for solving partial and differential equations, is investigated here for plate problem. A short overview is first given and the DQM formulation for the inclined cracked plate problem is presented.

3.1 An overview of the used DQM

The DQM requires to discretize domain of the problem into N points. The derivatives at any point are approximated by a weighted linear summation of all the functional values along the discretized domain, as follows [3]

$$\left.\frac{d^m f(x)}{dx^m}\right|_{x=\bar{z}_k}=\frac{d^m}{dx^m}\begin{bmatrix} f(\bar{z}_1)\\ f(\bar{z}_2)\\ f(\bar{z}_3)\\ .\\ .\\ .\\ f(\bar{z}_N)\end{bmatrix}=c_{kj}^{(m)}\begin{bmatrix} f(\bar{z}_1)\\ f(\bar{z}_2)\\ f(\bar{z}_3)\\ .\\ .\\ .\\ f(\bar{z}_N)\end{bmatrix} \quad (5)$$

where 'm' is the order of the highest derivative appearing in the problem, $f(\bar{z}_k)$ are the values of the function at the sampling points $\bar{z}_k$ relating the mth derivative to the functional values at $\bar{z}_k$. These coefficients can be determined by making use of Lagrange interpolation formula as follows:

$$f(x)=\sum_{i=1}^{N}\frac{L(x)}{(x-\bar{z}_i)L_1(\bar{z}_i)} \quad (6)$$

where: $L(x)=\prod_{j=1}^{N}(x-\bar{z}_j)$, $L_1(\bar{z}_i)=\prod_{j=1}^{N}(\bar{z}_i-\bar{z}_j)$

The weighting coefficients for the first order derivative to the functional values at $\bar{z}_k$ can be obtained as:

$$c_{kj}^{(1)}=\begin{cases}\dfrac{L_1(\bar{z}_i)}{(\bar{z}_k-\bar{z}_j)L_1(\bar{z}_j)} & k\neq j\\ -\displaystyle\sum_{j=1,\, j\neq k}^{N}c_{kj}^{(1)} & k=j\end{cases} \quad (7)$$

The second, third and higher derivatives can be calculated as:

$$c_{kj}^{(m)}=\sum_{l=1, j\neq k}^{N}c_{kl}^{(1)}c_{lj}^{(m-1)} \quad (8)$$

For accurate results, we adopt the Chebychev-Gauss-Lobatto mesh distribution given for interval [a,b] by:

$$\bar{z}_k=\frac{(b-a)}{2}[1-\cos(\frac{k-1}{N-1}\pi)] \quad k=1,2,...,N \quad (9)$$

It should be noted that this method has been used and deeply tested in case of multi-cracked beams [2,3].

3.2 Free vibration analysis using DQM

The free vibration problem is solved by using 2D differential quadrature method. By assuming a harmonic motion, the time-space partial differential equation (1) is reduced to the following partial differential equation :

$$\begin{aligned} D\nabla^4 w &= \rho H\omega^2 w(x,y)+\frac{CD(1+\cos(2\beta))}{((\alpha_{bt}/6)+\alpha_{bb})(3+\upsilon)(1-\upsilon)h+2C}\frac{\partial^4 w}{\partial y^4}\\ &+\frac{CD\upsilon(1+\cos(2\beta))}{((\alpha_{bt}/6)+\alpha_{bb})(3+\upsilon)(1-\upsilon)h+2C}\frac{\partial^4 w}{\partial x^2\partial y^2}\\ &+\frac{2CD\sin(2\beta)}{((C_{bt}/6)+C_{bb})(1+\upsilon)h+2C}\frac{\partial^4 w}{\partial x\partial y^3}\\ &+\frac{2CD\upsilon\sin(2\beta)}{((C_{bt}/6)+C_{bb})(1+\upsilon)h+2C}\frac{\partial^4 w}{\partial x^3\partial y}\end{aligned} \quad (10)$$

In order to use the DQM, the plane of the plate is discretized by an (n_e x m_e) Gauss-Lobatto-Chebychev grid. At a given point **(x_i ,y_j)**, of the plate, each derivative in Eq.(10) is written, in the DQM analog, as follows:

$$\left.\frac{\partial^p w(x,y)}{\partial x^p}\right|_{x=x_i}=\sum_{k_x=1}^{ne}c_{ik_x}^{(p)}w(x_{k_x},y_j) \quad (11\text{-a})$$

$$\left.\frac{\partial^q w(x,y)}{\partial y^q}\right|_{y=y_j}=\sum_{k=1}^{me}b_{jk_y}^{(q)}w(x_i,y_{k_y}) \quad (11\text{-b})$$

$$\left.\frac{\partial^{p+q} w(x,y)}{\partial x^p\partial y^q}\right|_{y=y_j}=\sum_{k_x=1}^{ne}c_{ik_x}^{(m)}\sum_{k_y=1}^{me}b_{jk}^{(m)}w(x_{k_x},y_{k_y}) \quad (11\text{-c})$$

After some mathematical developments and using a matricial form , the following eigenvalue problem is obtained

$$[K]\{w\}=\Omega^2\{w\} \quad (12)$$

After some mathematical developments and boundary conditions are incorporated so that:

$$\begin{bmatrix} A & A_b^t\\ A_b & 0\end{bmatrix}\begin{bmatrix}\{w\}\\ 0\end{bmatrix}=\Omega^2\begin{bmatrix}\{w\}\\ 0\end{bmatrix} \quad (13)$$

The matrix A_b is the matrix generating the boundary conditions. This resulting eigenvalue problem is to be solved; and {w} is a (n_e x m_e) row vector given by :

$$\{w\}^T=\{w(x_1,y_1)\ \ldots\ w(x_1,y_{me})\ \ldots..\ w(x_{ne},y_1)\ \ldots.w(x_{ne},y_{me})\} \quad (14)$$

The numerical solution of this eigenvalue problem allows getting the eigenfrequencies and eigenmodes of plates with various crack characteristics. The impact of both crack depth and orientation on natural frequencies and eigenmodes are investigated. These vibration characteristics will be used for cracks identification.

4. Cracks identification based on a corrected genetic algorithm

A genetic algorithm is a probabilistic search algorithm based on a model of natural evolution. As the population is generated randomly, a correction method is used here as well, based on the perturbed projected gradient to impose constraints to chromosomes. To estimate orientation (β), depth (h) and length (C) of the crack a multi-objective optimization based on PARETO method is adopted herein, where the following two fitness functions are used:

$$E = \sum_{i=1}^{N} \left(\left|\Omega_i - \omega_i\right|\right)^2 \qquad (15\text{-a})$$

$$\text{DLAC} = \frac{\{\partial\Omega\}^t \{\partial\omega_j\}}{\left(\{\partial\Omega\}^t \{\partial\Omega\}\right) \quad \left(\{\partial\omega_j\}^t \{\partial\omega_j\}\right)} \qquad (15\text{-b})$$

The DLAC (Damage Location Assurance Criterion) for a given individual j can be designed using a correlation approach, lying in the range of 0 to 1, with 0 indicating no correlation and 1 indicating an exact match between the patterns of frequency changes [1]. The individual j giving the highest DLAC value allows the best match to the measured frequency change pattern and is therefore taken as the predicted damage characteristics.

Ω and ω are the measured (experimental) and the theoretical eigenfrequencies respectively. Note that as previously discussed, the eigenfrequencies depend on crack orientation, depth and length, the more the eigenfrequency decreases, the more severe the damage is. A flowchart of implemented algorithm is given in Fig.2

5. Numerical results

For sake of brevity we present few results in this section. A square, simply supported plate having the following material properties: $E=7.03.10^{10}$N/m^2; $\rho=2660$ kg/m^3; $\upsilon=0.33$; Lx=Ly=1m; H=0.01mm. Table 1 presents the numerically obtained results based on DQM for various crack lengths. The crack is centrally located, and has a depth of h=0.05 and an orientation angle β=0. It is shown that numerical eigenfrequencies based on the DQM match well with semi-analytical ones based on multiple scales method. The plate is discretized into (13x12) grid points.

Table 1: Effect of crack length on the first eigenfrequency

	ω_1 (rad/s) SSSS-plate		
		Present	[6]
β=0	C=0 (Intact)	77.58	77.58
	C=0.01 m	75.5438	75.54
	C=0.025m	73.394	73.39

Other cases for β≠0 are investigated, cracks detection procedures are elaborated and the obtained results are well tested.

For the inverse problem, the proposed genetic algorithm is tested as well. Experimental eigenfrequencies are simulated by using ANSYS 14.0 software. Crack severity is evaluated through its parameters (h, C, and β). Predicted results are then compared to existing ones, to test efficiency of the proposed algorithm and its use for cracks identification in real situations.

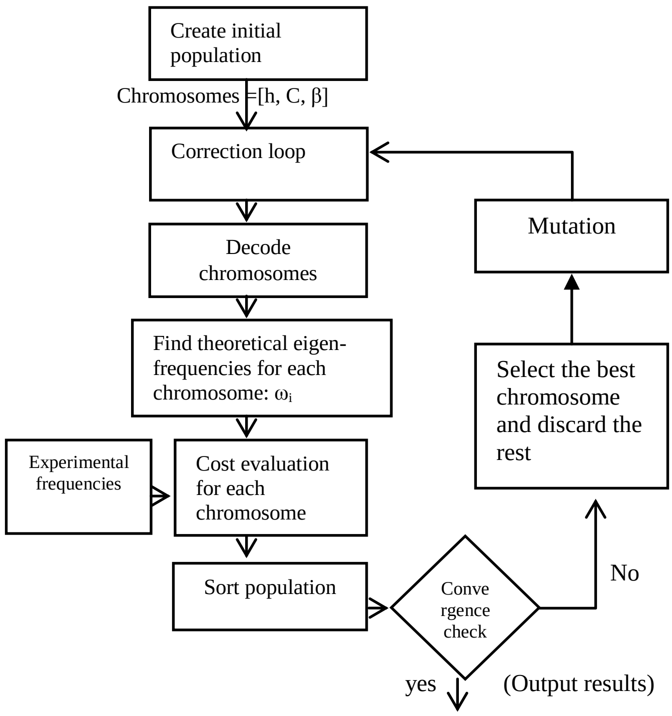

Figure 2: Flowchart of the implemented genetic algorithm

6. Conclusion

The Differential quadrature method is elaborated for free vibrations of variably oriented surface cracks in plates with different characteristics, and boundary conditions. Based on the obtained numerical results, inverse problem is solved for cracks identification. Combining corrected genetic algorithms with the investigated cracked plate model allow predicting crack parameters. Crack orientation, which affects significantly eigenparameters of the damaged plate, is also well detected according to this approach.

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