Plastic zone and effective distance under mixed mode fracture -Volumetric approach-

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Abstract:

The rupture mechanism assumes the presence of an initial defect, a place of stress concentration, after which the notch (case of our study) will be able to start.

It is therefore necessary to understand the mechanism of rupture and to delimit its volume of elaboration. This communication aims at locating and studying the critical size of this process zone, taking into account the loading conditions by examining them on steel specimens subjected to I + II mode in a volumetric approach.

This study aims to study the following axes:

1-Define the zone of elaboration of rupture by proposing new numerical methods.

2-Compare the extent of the plastic zone to the elaboration volume of the rupture.

To do this, two methods are developed: Stress gradient, plasticity criterion.

Keywords: *Volumetric approach, notch, effective distance, stress gradient, mixed mode, plasticity criterion, plastic zone*.

1. Introduction

Plastic deformation at a crack tip in material produces a plastic zone around the crack which keeps radially decaying stresses away from the crack tip. Various theoretical analyses have been reported over the last few decades to gain a comprehensive understanding of the mixed mode fracture, as done by Golos and Wasiluk[1], Erdogan and Sih[2] proposed a criterion based on the maximum tangential stress, Sih[3] presented a criterion based on the energy study, Theocaris et al.[4] used von Mises yield criterion for predicting the radius of plastic zone. But Irwin considered that the presence of a plastic zone at the bottom of crack, fact that the length of the crack behaves as if it was longer than its physical size and the stress distribution is equivalent to a crack elastic length (a+r), so its effective length , a_{eff} is :

$$a_{eff} = a + r_{eff} \qquad \text{With} \qquad R_p = 2\, r_{eff}$$

For simple estimation of the size of plastic zone along 'θ' equal to zero degree, considering a first approximation that plastic zone is circular with diameter Rp, for a perfectly elastic plastic material, according to:

$$R_p(\theta_0) = \lambda \left(\frac{K}{\sigma_V}\right)^2$$

With: σv stress tangential, K stress intensity factor and λ varies between 0.30 – 0.39 (Irwin: 0.318 and Dugdale [4]: 0.342).

2. Materials and Methodology

2.1 Material

The material studied is a high strength steel named 45CDS6 according to French standard. Mechanical properties are listed in next Table:

Table 1: MECHANICAL PROPERTIES OF 45CDS6 STEEL

E (MPa)	ν	σ_Y(MPa)	σ_U(MPa)	A%	Density (Kg/m³)	K_{IC} (MPa)
210065	0.28	1463	1662	2.8	7800	97

The microanalysis of the material givesthe following chemical composition:

Table 2: CHEMICAL COMPOSITION GIVEN IN ATOMIC PERCENTAGE

%C	%Mn	%Si	%Cr	%Mo
0.45	0.60	1.60	0.60	0.25

2.2Specimens

The experiments have carried out by examining various U-notched circular ring specimens (see Fig.1.), with various geometries and boundary conditions (Fig.2.).

With: external radiusRe=20mm, internal radius Ri=10mm, thickness B=7mm, and notch length a=4mm. Different notch radii are introduced using a wire-cutting electrical discharge machine and using wiresof different diameter. The notch root radius was measured using a profile projector. Five notchradius values are used: ρ = {0.15, 0.3, 0.5, 1, 2 mm}. With: 0° < α < 33° (mixed mode I+II). [6]

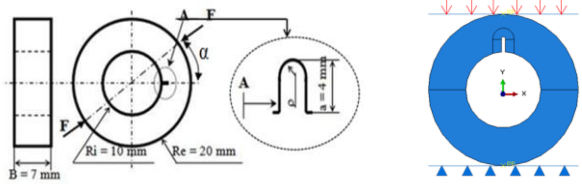


Figure 1: U-notched circular ring specimen and loading mode

The specimens are submitted to compression load in order to determine the critical loads when the fracture occurs. These loads are introducing to the simulation computation to finally evaluate the stress triaxiality evolution.

3. Finite Element Analysis

The nonlinear finite element simulations are performed usingABAQUS 6.10.The geometry of the U-notched circular was simplified by considering a plane part worthless thickness such as e=1, z=0.

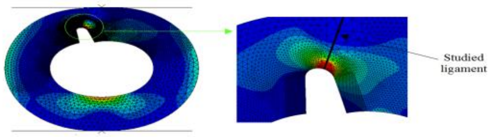


Figure 2: studied ligament

To study the transitional stages of mode I to the mode II, it's necessary to define new orientations of stress. It exist an angle corresponding to each mode of application of load namedθ_0. Method XFEM is very effective to study this kind of problem (applicable on Abaqus/CAE).
The following figure summarizes the values of the angles drawn on (Abaqus/CAE) and raised.

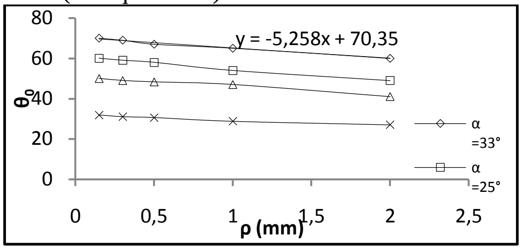


Figure 3:Fracture angle θ vs radii ρ

The numerical values obtained according to θare compared to the experimental values measured by an optical microscope.In mode II (for α=33°), for a radius notch ρ=0:θ_0= 70,35°
The evolutions of stress with various radii are presented in the sections at the bottom, in mixed-mode (I+II) fracture crack initiation from notches is governed by the tangential stress.The stress evolutions areplotted versus the notch tip distance for each angle α, for (ρ=1).The maximum stress values decrease when the notch radius increases.

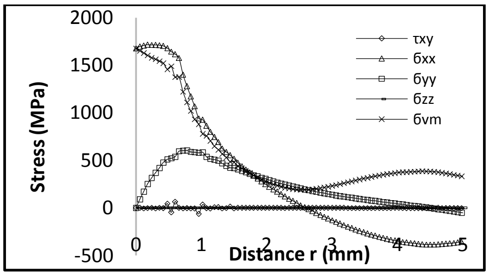


Figure 4: stress distribution vs distance r

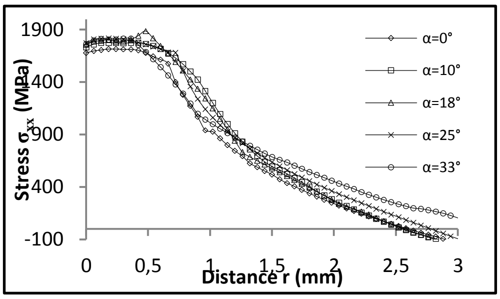


Figure 5: stress-the different angles

4. EFFECTIVE DISTANCE AND VOLUMETRIC METHOD

The volumetric method is a local fracture criterion, which supposes that the fracture process requires a certain fracture volume. This volume is assumed as a cylinder with effective distance at its diameter. Three distinct zones in the diagram can be distinguished:
- Zone I: the elastic-plastic stress opening stress increases and attains a peak value.
- Zone II: the elastic-plastic stress drops gradually in the elastic regime.
- Zone III: starts at a certain distance which is named the effective distance. It represents linear behavior in the bi-logarithmic diagram.

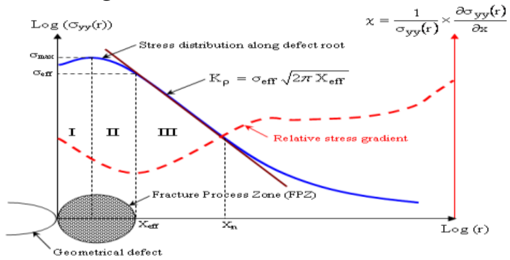


Figure 6: schematic elastic-plastic distribution

The notch stress intensity factor is defined as the function of effective distance and effective stress:
$K\rho = \sigma_{eff}\sqrt{2\pi X_{eff}}$
By definition, the effective distance is the diameter of the process volume assuming it has a cylindrical shape. This function represents a minimum corresponding to the effective distance X_{eff}:$\chi(r) = \frac{1}{\sigma_{xx}(r)}\frac{d\sigma_{xx}(r)}{dr}$
Where: $\chi(r)$ and $\sigma_{xx}(r)$, are relative stress gradient and maximum principal stresses or crack opening stress, respectively.
Average volume of the stress distribution over the effective distance. However stresses are multiplied by a weight function in order to take into account the influence of stress gradient due to geometry and loading mode. The effective stress is defined as:
$$\sigma_{eff} = \frac{1}{X_{eff}}\int_0^{X_{eff}} \sigma_{xx}(r)(1 - r\chi(r))dr$$

5. VON-MISES STRESS AND PLASTIC RADIUS

The criterion of plasticity Rp is defined by the point of intersection of the stress Von Mises and the yield stress of material.

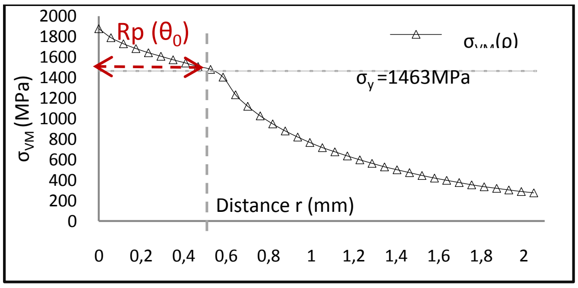
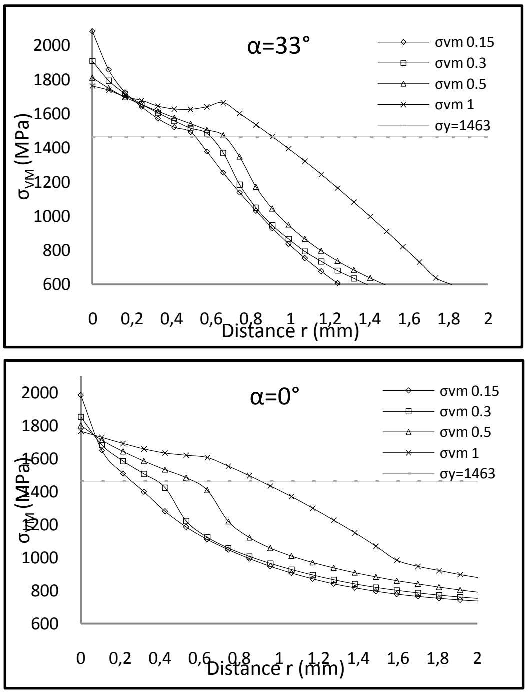


Figure 7: Determination of the plastic radius by stress Von Mises

This model of behavior of criterion of plasticity considers that the threshold from which the plastic flow develops, the constraint equivalent to the zone of development of rupture is in intersection with the yield stress (with σ_y=1463MPa).The notch generates a concentration to the front of defect, which involves a fall of the resistance of the U-notched circular ring specimens. There is then risk of rupture.
The examination of shows that the extent of the plastic zone at the bottom of notch varies according to the values of bifurcation angle.
This figure shows the distribution of von mises stress for different angles of notch: 0and 33°. The change in parameters of notch, radius and angle, changes the morphology of plastic zone near the bottom of notch.



Figures8-9 : distribution of Von mises

6. RESULTS AND DISCUSSIONR_p AND $R_p^{\rho}(\theta_o)$ ANALYTICAL

The parameters introduced into the elastoplastic zone $R_P(\theta_o)$defined by Irwin, (1) are valid only in case of a crack (ρ=0 and ψ=0). After results analysis and for a perfectly elastoplastic materialaccording to a rupture under notch effect,$R_p^{\rho}(\theta_o)$can be proposed and written in the following form:

$$R_p^{\rho}(\theta_o) = \frac{1}{2\pi}\left(\frac{K_\rho}{\sigma_{eff}}\right)^2$$

For critical loads, the expression will be written:

$$R_p^{\rho c}(\theta_o) = \frac{1}{2\pi}\left(\frac{K_\rho^c}{\sigma_{eff}^c}\right)^2$$

With: K_ρ^c critical stress intensity factor, σ_{eff}^c critical effective stress.

By the analytical method and, we calculate the values of$R_p^{\rho c}(\theta_o)$, and its well be compared with the values of Rp by Von Mises yieldcriterion. The table 3 summarizes the various values of RpandR$_p^{\rho}(\theta_o)$.

Table 3: comparison of plastic zone by Von Mises

Mode	ρ	R_p (mm)	$R_p^{\rho c}(\theta_o)$ (mm)	$\frac{R_p}{R_p^{\rho c}(\theta_o)}$
α=0°	0,15	0,585	0,64	0,91
	0,3	0,6004	0,6	1
	0,5	0,57	0,66	0,86
	1	0,5401	0,73	0,74
	2	0,84	1,03	0,81
α=33°	0,15	0,312	0,34	0,91
	0,3	0,436	0,56	0,78
	0,5	0,595	0,75	0,79
	1	0,825	0,8	1,04
	2	1,433	1,39	1,03

This table shows the evolution of the plastic zone by the two methods (analytical and by the constraint of Von Mises) for various specimens. It is shown clearly that the distribution varies with a weak variation, but the values are very close, and the results are in agreement.

$$\frac{R_p}{R_p^{\rho c}(\theta_0)} \approx 1 \qquad \rightarrow \qquad Rp \approx R_p^{\rho c}(\theta_o)$$

7. R_p AND EFFECTIVE DISTANCE

To interpret the various criteria for Rpand their convergence towards the determination of the effective distance X_{eff} to the bottom of the notch, the table 4 summarizes all those values.

Table 4: comparison of plastic radius Rp with effective distance Xeff

Mode	ρ	X_{eff}	R_p (mm)	X_{eff}/R_p
α=0°	0,15	0,585	0,585	1
	0,3	0,6004	0,6004	1
	0,5	0,6603	0,57	1,2
	1	0,7202	0,5401	1,3
	2	1,0199	0,84	1,2
α=33°	0,15	0,312	0,312	1
	0,3	0,517	0,436	1,2
	0,5	0,694	0,595	1,2
	1	0,734	0,825	0,9
	2	1,283	1,433	0,9

The examination of the evolution of the various values shows that the size of the plastic zoneincreases with the radius of notch.The analyses above, assume that the zone of fracture process is larger than the plastic zone.

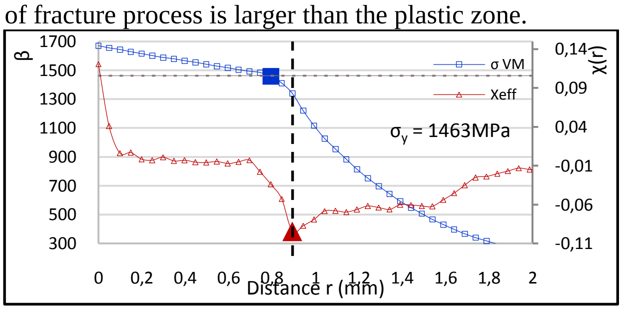


Figure10: comparison Xeff by Rp

In both cases, the values of X_{eff} are considered to be larger than the plastic zone; the fracture process zone is covering the plastic zone.

8. CONCLUSION

In order to evaluate the elastoplastic zone, the modeling of a problem is carried out by using two methods to describe it; by using Von Mises stress andby analytical method; good agreement between the two forms of the plastic zone were be achieved. Furthermore, the result of analysis shows that the effective distance X_{eff}may be determined by taking into account the following principles:-The effective distance X_{eff}can be determined accepting that this distance is supposed to be larger than the plastic zone diameter.-The stress gradient envelops the criterion of plasticity. Moreover, the calculation of the size of the plastic zone depends mainly on the type of specimen, the radius and angle of notch too.

REFERENCES

[1] Golos, K., &Wasiluk, B. Role of plastic zone in crack growth direction criterion under mixed mode loading. *International journal of Fracture*, 102(4), 341-353, 2000.

[2] Shih, C. F., &Asaro, R. J. Elastic-plastic analysis of cracks on bimaterial interfaces: part I—small scale yielding. *Journal of Applied Mechanics*,55(2), 299-316, 1988

[3] Sih, G. C. Strain-energy-density factor applied to mixed mode crack problems. *International Journal of fracture*, 10(3), 305-321, 1974.

[4] Dugdale.S. "Yielding of steel sheets containing slits", *Journal of Mechanical Physics of Solids*, vol8, pp 100-104, 1960.

[5] M. Moussaoui, S. Meziani. Study of the Evolution of Elastoplastic Zone by Volumetric Method. *International Journal of Mechanics and Applications*, 2015

[6] O. Zebri, H. El Minor, A. Ben Darma. Evaluation of the effective distance by stress triaxiality on mixed mode fracture (Volumetric approach). *International Review of Mechanical Engineering. 2016*