Mixed convection in lids driven rectangular cavity field with Newtonian fluid and laterally heated

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Abstract:
The present combined analytical and numerical work explores the mixed convection in a lengthened rectangular cavity of aspect ratio, A = 24, confining a Newtonian fluid of Prandtl number, Pr = 7, this study is made in two cases, in the first we consider the upper and the lower wall are mobile, in the second case we accept that only the upper wall is moving. Additionally, the horizontal walls are supposed adiabatic and the vertical ones are subjected to a uniform heat flux density.

Keywords: *Mixed convection, lids-driven cavity, parallel flow, finite volume method, heat transfer*

1. Introduction

The study of mixed convection heat transfer mechanism in lids driven cavity has received enormous attention due to its pragmatic as well as theoretical significance, since the details of flow and heat transfer insights of cavity as well as Nusselt number (or dimensionless heat transfer coefficient) are quite useful for possible applications in various fields such as, design of cooling systems of electronic gadgets, high performance building insulation, multi-shed structures, furnace, food processing, lubrication technologies, solar heat collectors, drying, etc. [1], [2], [3] and [4].

However, previous work on the study of this phenomenon essentially referred the case where the cavity is square [4-5]. The case of a horizontal cavity, which has not too attracted interest from the scientific community, may reveal different results such as shown in the study already conducted in this direction if all its walls still [6].This study addresses a rectangular cavity in both cases, in the first we consider the upper and the lower wall are mobile, in the second case we accept that only the upper wall is moving. Furthermore, the horizontal walls are supposed adiabatic and the vertical ones are subjected to a uniform heat flux density.

This issue is governed by a system of nonlinear partial differential equations, expressed in terms of horizontal and vertical components of velocity, temperature and pressure. With the approximation of a parallel flow, an analytical solution is obtained when the aspect ratio is wide. The governing equations are also solved numerically using an approach based on the finite volume method in a regular grid and that in order to demonstrate the validity of analytical results

2. Mathematical formulation:

The physical system under study is sketched in Fig.1. It basically consists of a double-lid-driven rectangular cavity with height H' (y) and length L' (x), filled with a Newtonian fluid. The bottom and top lids impart a steady sliding motion in the opposite direction sharing the uniform velocity, U_0. The long horizontal walls are adiabatic, while the vertical short ones are sub- mitted to a uniform density of heat flux, q'.

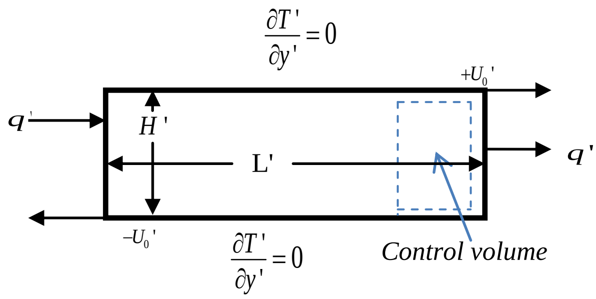

Figure 1 : schematic view of the geometry and coordinates system

We accept the following assumptions:

- The fluid is Newtonian and incompressible obeys the Boussinesq approximation
- The flow is two-dimensional, laminar and steady
- The radiation heat transfer between the sides of the cavity is negligible when compared with the other mode of heat transfer

The governing equations and the corresponding boundary conditions are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial P}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial P}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+RiT \quad (3)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\operatorname{Re}\Pr}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \qquad (4)$$

$$u=v=\frac{\partial T}{\partial x}=0 \quad \text{For x=0 and A} \qquad (5)$$

$$u+1=v=\frac{\partial T}{\partial y}=0 \quad \text{For y=0} \qquad (6)$$

$$u-1=v=\frac{\partial T}{\partial y}=0 \quad \text{For y=1} \qquad (7)$$

Where: $A=\frac{L^{'}}{H^{'}}$, $Pe=\frac{H^{'}U_0}{\alpha}$, $Re=\frac{H^{'}U_0}{(K/\rho)}$,

$Ri=\frac{g\beta q^{'}H^{'2}}{\lambda U_0^{'2}}$, $\Pr=\frac{K/\rho}{\alpha}$

Note that $Pe=\Pr\operatorname{Re}$ and $Ri=\frac{Gr}{\operatorname{Re}^2}=\frac{Ra}{Pe\operatorname{Re}}$

The local heat transfer, in the horizontal direction of the cavity, can be expressed in terms of the local Nusselt number defined by the follow equation:

$$\overline{Nu}=\int_0^1-\frac{1}{(\partial T/\partial X)_{X=A/2}} \qquad (8)$$

3. Numerical method

In order to solve the governing partial differential equations along with the boundary conditions we have used the finite volume method of Patankar [7] and SIMPLER algorithm in a staggered uniform grid system. The choice of this technique lies in the advantages it offers in terms of numerical stability, convergence and conservation of flows on each elementary volume.

The choice of the mesh grid depends on the aspect ratio, A, of the cavity and also the nature of the solution. Numerical tests were required to optimize the time and accuracy of the calculations. Thus the uniform grid of 381 × 81 is found sufficient to accurately model the flow and temperature fields in a cavity having A = 24.

4. Approximation of parallel Flow

The case of double-lid-driven cavity

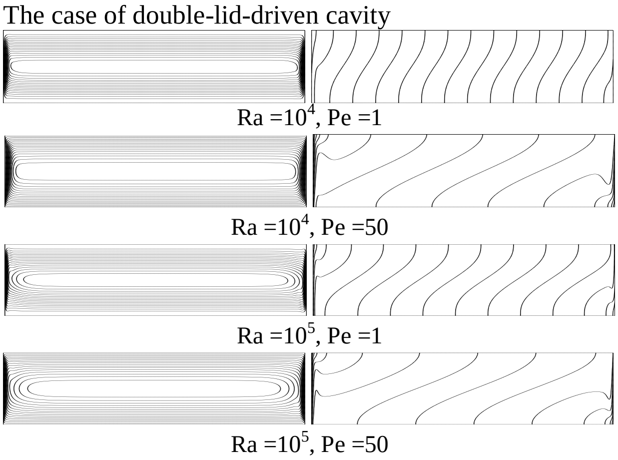

Figure 2 : streamlines (left) and isothems (ritght) for Ra (10^4 and 10^5) and for Pe (1 and 50) in double-lid-driven cavity

The case of a lid-driven cavity

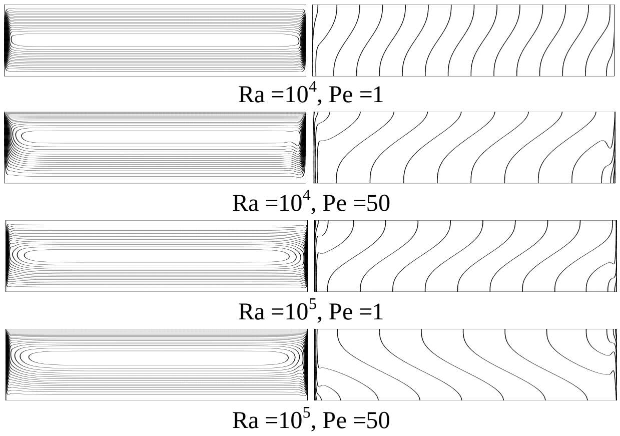

Figure 3: streamlines (left) and isothems (ritght) for Ra (10^4 and 10^5) and for Pe (1 and 50) in a lid-driven cavity

In the both cases (a lid-driven cavity and double-lid-driven cavity), it is interesting to note that the flow is unicellular turning in clockwise, as a result of the cooperative aspect of the shear and buoyancy effects that work together from left to right

In above Figures (2-3), the flow and temperature fields exhibit a parallel aspect and a linear stratification, respectively, in the most part of the cavity, there proves the existence of an analytical solution.

$$u(x,y)=u(y)=\frac{d\psi}{dy},\ v(x,y)=-\frac{d\psi}{dx}=0 \qquad (9)$$

And $T(x,y)=C\left(x-A/2\right)+\theta(y)$ (10)

Where C is the horizontal gradient of Temperature can be obtained by:

$$C+1=\operatorname{Re}\Pr\int_0^1 u\theta\, dy \qquad (11)$$

The equations (9-11) can be rewrite

$$\frac{d^3u}{dy^3}=\operatorname{Re}RiC \quad (12), \qquad \frac{1}{\operatorname{Re}\Pr}\frac{d^2\theta}{dy^2}=Cu \quad (13)$$

With: $u+v_0=v=\frac{\partial T}{\partial y}=0$ For y=0 and

$$u-1=v=\frac{\partial T}{\partial y}=0 \quad \text{For y=1} \qquad (14)$$

$$\int_0^1 u(y)dy=0 \quad (15), \qquad \int_0^1 \theta(y)dy=0 \quad (16)$$

The solution of the equations (12) and (13), satisfying (14)-(16), is:

$$\begin{aligned}\mathrm{u}(y)=&\frac{1}{12}\operatorname{Re}RiC\left(2y^3-3y^2+y\right)+\left(3y^2-2y\right)\\&+v_0\left(3y^2-4y+1\right)\end{aligned} \qquad (17)$$

$$\begin{aligned}\psi(y)=&\frac{1}{12}\operatorname{Re}RiC\left(\frac{y^4}{2}-y^3+\frac{y^2}{2}\right)+\left(y^3-y^2\right)\\&+v_0\left(y^3-2y^2+y\right)\end{aligned} \qquad (18)$$

$$\theta(y)=\frac{1}{12}\Pr Ri\,Re^2 C^2\left(\frac{y^5}{10}-\frac{y^4}{4}+\frac{y^3}{6}-\frac{1}{120}\right)+Re\Pr C\left(\frac{y^4}{4}-\frac{y^3}{3}+\frac{1}{30}\right)+v_0\,Re\Pr C\left(\frac{y^4}{4}-2\frac{y^3}{3}+\frac{y^2}{2}-\frac{1}{20}\right) \quad (19)$$

The transcendental equation is:

$$1+\left(1+\frac{Re^2\Pr^2}{105}-\frac{v_0\,Re^2\Pr^2}{70}+\frac{v_0^2\,Re^2\Pr^2}{105}\right)C+\left(\frac{v_0\,Re^3\Pr^2 Ri}{3360}-\frac{Re^3\Pr^2 Ri}{3360}\right)C^2+\frac{Ri^2\,Re^4\Pr^2}{362880}C^3=0 \quad (20)$$

The heat transfer becomes: $\overline{Nu}=-\frac{1}{C}$ (21)

It is to highlight that if $v_0=0$ this is the case of a lid-driven cavity but for the double-lid-driven cavity v_0 receives the value -1.

5. Results and discussion:

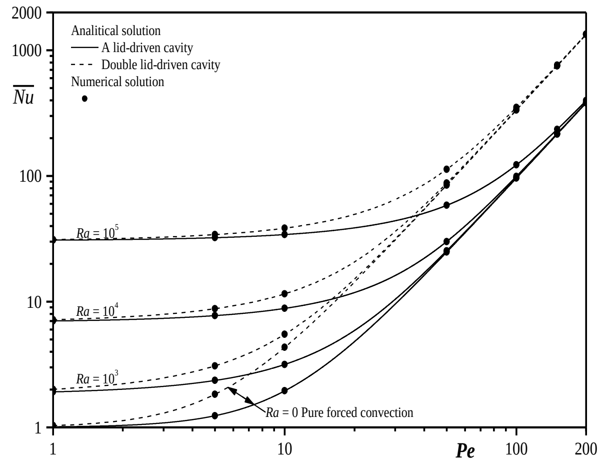

Figure 4: Heat transfer rate versus Pe for various values of Ra

Figure 4, where it has postponed the change in the average Nusselt number depending on the Peclet number, shows that for a fixed Rayleigh number, heat transfer is almost constant for low values of Peclet, follows it increases relatively slowly and eventually grows very quickly. This means that the shearing effect improves the heat transfer.

Rayleigh number effect on the heat transfer is illustrate in figure 5, for low Peclet number values the heat transfer variation is almost linear, then increasing of the Peclet number the variation becomes relatively slow until it will be unchangeable. The increase of the Rayleigh number gives great values for the heat transfer because the buoyancy becomes, gradually, the main driving force for the fluid motion.

In other hand the heat transfer values in double-lid-driven cavity are relatively higher than those obtained in a lid-driven cavity, so we can say that the movement of the walls has a positive effect on the heat transfer

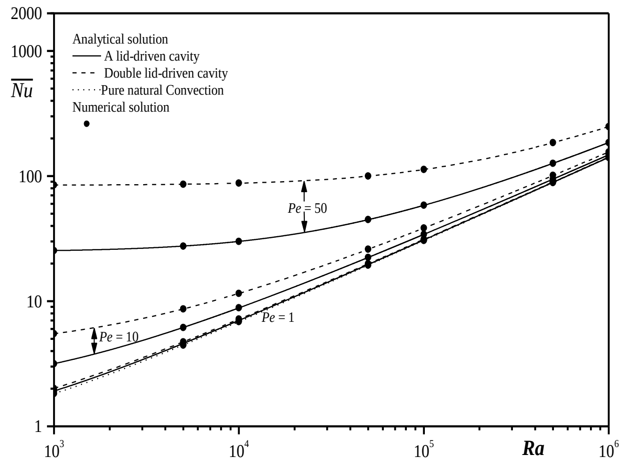

Figure 5: Heat transfer rate versus Ra for various values of Pe

6. Conclusion

1. The analytical results agree very well with the numerical ones which validate the approximation of the parallel flow and the numerical code.
2. The Peclet, Pe, and the Rayleigh, Ra, numbers have positive effect on the mean heat transfer
3. The movement of the walls promotes the heat transfer

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