

Two-dimensional modeling of flow in variably saturated porous media with the capillary fringe

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Abstract

Recent studies have demonstrated the importance of the capillary fringe in hydrological and biochemical processes. In addition, the role of the capillary fringe in shallow waters is likely to be significant. Traditionally the study of groundwater flow focus on unsaturated-saturated zones without taking the influence of the capillary fringe in consideration. In this study, We have made a clear and precise definition of the capillary fringe and proposed a 2-D steady-state solution. The model is based on the finite element method using the Freefem++ code and generates solution for two-dimensional water flow. The validity of the model is proved by the excellent agreement between simulated and experimental data measured in the laboratory-scale physical model. In general, the numerical model constitutes a rapid tool to predict with an excellent precision the water flow.

Keywords: *Capillary fringe , modeling, porous media*

1. Introduction

Generally, it is considered the transition zone between the unsaturated and the saturated zones, the lower boundary being the groundwater table, at which the water and the atmospheric pressures are identical. The upper boundary is less clearly defined: a rigorous definition limits the capillary fringe to the water-saturated zone directly above the water table, where the water pressure is smaller than the atmospheric pressure due to capillary forces, whereas an extended definition of the capillary fringe includes the tension-saturated zone and the contiguous variably saturated parts above. Horizontal flow above the water table has been reported in some experiments (e.g., Silliman et al., 2002) and numerical models (Boufadel et al., 1999;). Textbooks on groundwater hydrology virtually ignore the role of the capillary fringe in flow and transport. Except near regions of discharge to surface water bodies, fluid flow

and chemical transport in the subsurface are usually conceptualized as primarily vertical in the unsaturated zone, with transition to fully three-dimensional flow below the water table (Berkowitz, 2004).

Its impact on the physical and biochemical activities also depends on soil types and pore sizes (sand, gravel,...). The structural types of CF that can arise from a number of competing pore-scale displacement mechanisms. Except some studies (Ronen, 1997) considering the relative water content as the upper limit or including the saturation and near-saturation zones above the water table, there is a general consensus on the definition of the capillary fringe, regarded as the zone above the water table where all the pores are fully saturated with a negative pressure head (Silliman 2002). FreeFem++ also takes advantage of the multicore processors structure and parallel computing. While The capillary fringe has been considered in different natural situations like coastal beaches and estuaries there are less descriptions for two dimensions unsaturated-saturated model.

2. Study domain

In the laboratory experiments, a slab of soil with a length of 300 cm and 200 cm of depth was used. A rainfall simulator was designed as a source of flow in an infiltration band of 100 cm (Iemacha et al, 2006). To impose a constant hydraulic head in the outlet, a reservoir was attached to the slab. The soil is assumed to be homogeneous, rigid and isotropic.

We consider the coordinate system xOz , the Ox axis correspond to the surface of the ground, the Oz axis being positively oriented down and point O being in the infiltration band centre. The soil surface is supposed to be flat. We assume the symmetry about the Oz axis and the problem as a plane one; the problems of transfers can be studied only in the half of the system. Assuming a horizontal water table established on a thickness e above an impermeable bottom at a depth of e_0 , we apply to the

surface of the ground a constant infiltration flow of q_0 .

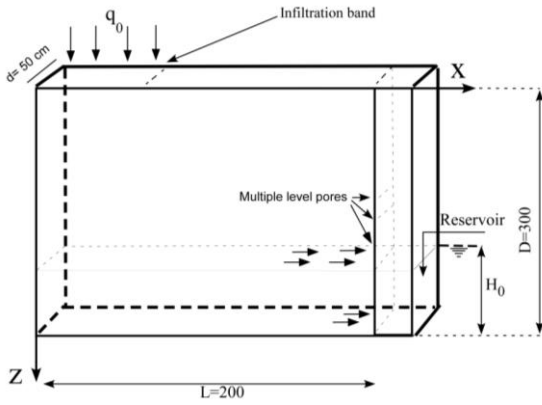


Fig. 1 Schematic representation of the flow domain.

Furthermore, we assume that reservoir is between two parallel trenches of the same length as the slab, where the piezometric level is maintained at a constant depth. Figure 1 shows the geometry of the studied system. The flow domain boundaries are:

- A portion of the upper horizontal surface (soil surface) on which is applied an initial flux $q_0 = 15 \text{ cm}^2/h$, causing infiltration. The infiltration band will simulate a canal of irrigation or an artificial catchment basin.
- Equidistant trenches of the axis of the infiltration band where we can impose a body of water that is limiting the thickness of the water table. Each trench can simulate a drainage ditch.

3. Physical model

Gardner model:

The Gardner hydraulic conductivity function, modified by Philip to accommodate a non-zero air-entry pressure (he) is:

$$K(h) = \begin{cases} K_s \exp(\alpha_G(h - h_e)) & \text{for } h \leq h_e \\ K_s & \text{for } h > h_e \end{cases}$$

where α_G [L^{-1}] is a constant. It used this form in formulating the analytic element method for soils of differing α_G .

The water content $\theta(h)$ is expressed by:

$$S_e(h) = \begin{cases} \frac{\theta_e(h) - \theta_r}{\theta_{es} - \theta_r} = \exp(\beta_G(h - h_e)) & \text{for } h \leq h_e \\ \frac{\theta(h) - \theta_r}{\theta_s - \theta_r} = 1 & \text{for } h > h_e \end{cases}$$

The Gardner function, even with its limited parameter set, has been shown to fit hydraulic conductivity data well over a limited range of pressures. The main limitation on using the Gardner function is a lack of flexibility to represent soils over the full pressure range.

Kirchoff potential:

Neglecting the compressibility of water, the movement of water in the unsaturated soil is based on the 2D Richards equation:

$$C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left(K(h) \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial z} \left(K(h) \left(\frac{\partial h}{\partial z} - 1 \right) \right)$$

To have a potential form of Richards equation, we use the Kirchoff integral transformation, defined below, on (3):

$$\varphi(h) = \varphi_{ref}(h) + \int_{ref}^h K(s) ds \quad \text{for } h \leq h_e$$

With $\varphi(h)$ [$L^2 T^{-1}$] the Kirchoff potential, φ_{ref} the referential potential often taken null for calculation facilities:

$$\varphi_1(h) = \int_{-\infty}^h K(s) ds \quad \text{for } h \leq h_e$$

At any point the flux may be regarded as the sum of a matric component given by the gradient of φ and a gravitational component. Therefore, it is appropriate to name φ the matric flux potential.

We use the Leibniz's rule for differentiation with one variable:

$$\frac{\partial \varphi_1(h)}{\partial x} = K(h) \frac{\partial h}{\partial x} \quad \text{and} \quad \frac{\partial \varphi_1(h)}{\partial z} = K(h) \frac{\partial h}{\partial z}$$

Combining equations (3), (6) and (7), we obtain the potential form of the Richards equation characterizing the steady state flow in the unsaturated zone:

$$\frac{\partial^2 \varphi_1(h)}{\partial^2 x} + \frac{\partial^2 \varphi_1(h)}{\partial^2 z} + \alpha_G \frac{\partial \varphi_1(h)}{\partial z} = 0$$

Discharge potential

We will determine the equation governing the flow according to the discharge potential in the capillary fringe and the saturated zone. In fact, the same equation governs the two zones, it's justified by the saturated nature of the pores in the capillary fringe. (Luthin and Miller 1953) indicated that the Laplace equation can be used to analyze the flow pattern in the capillary fringe. Therefore, the problem is posed in terms of discharge potential (Wong JR S and Craig, 2010):

$$\varphi_2 = K_s(h + z)$$

Darcy's law expresses relates the volumetric flux \vec{q} [LT^{-1}] to the hydraulic head H [L] according to:

$$q = -K_s \frac{\partial H}{\partial z} = -\frac{\partial(K_s H)}{\partial z}$$

thus, in combination with the mass conservation law:

$$\text{div}(\vec{q}) = 0 \quad \text{and} \quad \vec{V} = \overrightarrow{\text{grad}}(\varphi_2)$$

$$\text{implies: } \frac{\partial^2 \varphi_2(h)}{\partial^2 x} + \frac{\partial^2 \varphi_2(h)}{\partial^2 z} = 0$$

4. Results and discussion

Figure 2 shows the profile of the simulated water table by the numerical model and the measures at the steady state reached after 8 hours. We note that the results are consistent with a very low error rate.

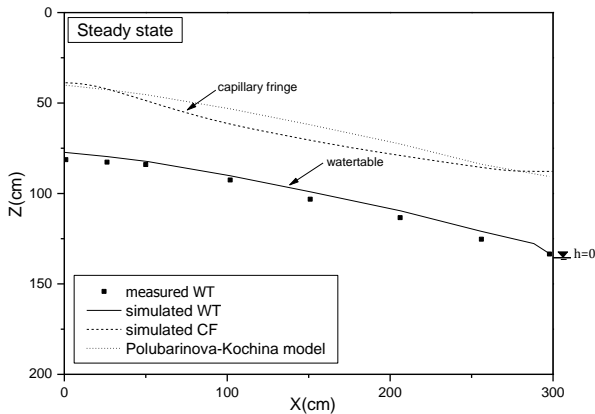


Fig.2. The measured and simulated level of water table and the two capillary fringe upper limit models

Figure 3 shows rather good agreement between the two models. The fixed value of the Polubarinova-Kochina model miss to represent the changes of the capillary fringe height specially in the middle zone.

Near the outlet, there is an irregularity in the pressure head curve probably due to the absence of a seepage surface in our made model, forcing the pressure head to move quickly to zero at the exit. The model captures the observed experimental behavior fairly well.

Performance analyzes

The performance of our model has been tested by statistical indicators (Loague 1991). The root mean square error (*RMSE*) was compared to the mean measured value and the normalized root mean square error (*NRMSE*) was used as a measure of the goodness of the fit. They are defined as follows:

$$RMSE = \sqrt{\left[\frac{\sum_{i=1}^n (Z_{meas_i} - Z_{pred_i})^2}{N} \right]}$$

$$NRMSE = \left[\frac{RMSE}{Z_{meas}} \right]$$

The uncertainty of the model prediction has been estimated, the results demonstrate the good performance of the model and confirms the water table level prediction.

At the steady state, the water table stabilizes to a maximum position.

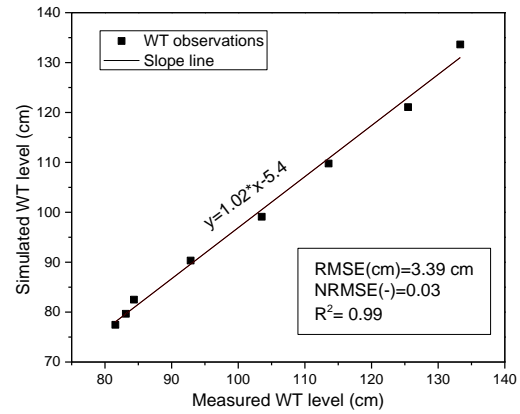


Fig.3. Statistical indicators of the hydrodynamic model performance

The domain is hydro-statically balanced and therefore the flow entering through the infiltration band becomes equal to that exiting through the outlet of the aquifer. The figures confirm that the velocity of flow is in the direction of potential decrease.

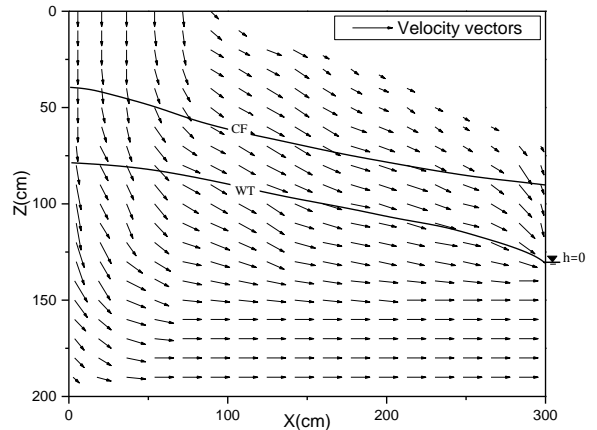


Fig.4 Simulated flow vectors

The figures confirm that the velocity of flow is in the direction of potential decrease. The behavior of vectors in the capillary fringe can be divided into three zones.

Conclusion

The hydrological studies by the potential method was given less intention in favor of head and pressure methods. As presented in this paper, it remains appropriate and opportune to focus our studies on the potential form of *Richards* equation. The combination of the discharge potential and Kirchhoff potential can be used satisfactorily in ground water dynamics.

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