WIND TURBINE BLADE OPTIMISATION WITH AXIAL INDUCTION FACTOR AND TIP LOSS CORRECTIONS

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Abstract

Tip loss corrections are a critical factor in blade element momentum theory when determining optimum blade shape for maximum power production. Using numerical and analytical optima, this paper compares the optimal tip shape using the classical tip loss correction of Glauert. A semi-analytical solution was proposed to find the optimum rotor considering Shen's new tip loss model. The optimal blade geometry is obtained for which the maximum power coefficient is calculated at different design tip speed and glide ratio. Our simulation is conducted four S809 rotor wind turbine blade type, produced by National Renewable Energy Laboratory (NREL).

Keywords: *BEM method, S809 airfoil, Horizontal axis wind turbine, Aerodynamic performances, Tip speed ratio, power coefficient*

1. Introduction (12 gras)

Optimisation of wind turbine rotors frequently involves the use blade element momentum (BEM) models both analytically, or with numerical methods. These numerical methods can be used where no analytical optimisation is known, as well as maximising two or more objective functions. State-of-the-art BEM models are based on two-dimensional flow models and hence are limited in their ability to accurately determine three-dimensional flow structures. Whilst Navier-Stokes solvers can be incorporated into rotor optimisation for greater accuracy, BEM computations have significant advantages in computational speed and ease of implementation. An alternative to using a more computationally intensive method is to modify the BEM model and apply corrections to the technique. One of the most important corrections to BEM analysis is a tip loss correction. The concept of a tip loss was introduced by Prandtl [4] to account for the difference between an actuator disc with an infinite number of blades and a real wind turbine or propeller with a finite number of blades. Glauert [1] developed blade element-momentum theory based on one-dimensional momentum theory as a simple method to predict wind turbine or propeller performance. In order to make more realistic predictions, Glauert introduced an approximation to Prandtl's tip loss correction to be included in BEM computations. In his analysis, Glauert assumed that the tip loss only affected the induced velocities but not the mass flux. Later, de Vries [2] corrected both the induced velocity and the mass flux. Recently, Shen et al. [3] showed existing tip loss corrections to be inconsistent and argued that they fail to predict correctly the physical behavior in the proximity to the tip. Shen et al. introduced a new tip loss correction model that gave better predictions of the loading in the tip region.

Numerical methods may also be useful with the inclusion of the so called Glauert empirical correction. If the axial induction factor, a, reaches values greater than 0.5, momentum theory is no longer holds and it is common to correct the local coefficient of thrust, C_T, using a simple linear relationship. To avoid confusion between the 'Glauert empirical correction' and 'Glauert's tip correction' the phrase 'high thrust modification' has been used throughout this manuscript.

Shen et al. [3] used a value of a_c = 1/3 which was followed here.

In theory, for a rotor operating at the Betz limit, a_c = 1/3 and hence uncorrected momentum theory should hold. Further, it is arguable whether the high thrust modification should be used in wind turbine power computations.

The objective of the current work is to maximise the power extraction efficiency of a wind turbine rotor, with a focus on the the effect of various tip loss models on the optimal rotor shape.

2. Mathematical model

As the classical theory of wind turbine rotor aerodynamic, the BEM method combines the momentum and blade element theory. The blade is divided into several elements as shown in Fig. 1, by applying the equations of momentum and angular momentum conservation, for each element dr section of the blade, axial force and torque can be defined by Eqs. (1) and (2), respectively.

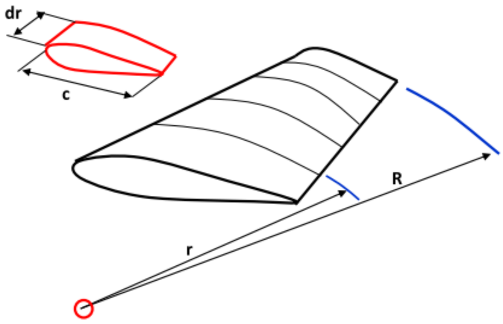

Fig.1: Blade element model

$$dT = \frac{1}{2} B\rho c V_{rel}^2 C_n dr \quad (1)$$

$$dM = \frac{1}{2} B\rho c V_{rel}^2 C_t r dr \quad (2)$$

Here, ρ is the air density, B is the number of blades, c is the chord length, V_{rel} is the relative wind speed, r is the local radius, Ω is the rotor angular velocity.

The relative wind speed, V_{rel}, is given as follow:

$$V_{rel} = \sqrt{V_0^2(1-a) + (r\Omega(1+b))^2} \quad (3)$$

These relations from the momentum theory alone do not include the effects of blade shape, for it, the blade element theory was introduced. For the airfoil section of a horizontal axis wind turbine shown in Fig. (2).

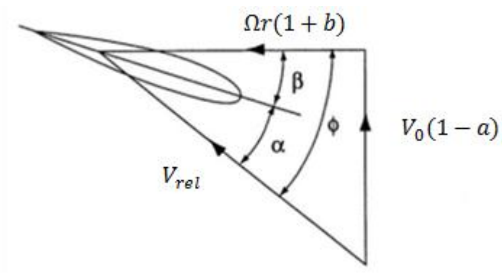

Fig.2: Cross-sectional airfoil element

Accordingly, the angle of relative wind, ϕ, is determined by:

$$\tan(\phi) = \frac{(1-a)V_0}{(1+b)\Omega r} \quad (4)$$

The optimal twist angle given by:

$$\beta_{opt} = \phi - \alpha_{opt} \quad (5)$$

Where α_{op}, is the optimal angle of attack, it extracted from 2D CFD calculation.

The turbine has B number of blades. Therefore, the force of thrust and torque at each element *dr* are given by Eq. (6) and (7) respectively:

$$dT = 4\pi B\rho V_0^2 a(1-a) r dr \quad (6)$$

$$dM = 4\pi B\rho V_0 \Omega b(1-a) r^3 dr \quad (7)$$

Moreover, λ_r is the local tip speed ratio as defined by:

$$\lambda_r = \frac{r\Omega}{V_0} = \frac{r\lambda}{R} \quad (8)$$

By equating (1) with (6) and (2) with (7), the axial and tangential induction factors can be found as follows:

$$a = \frac{\sigma C_n}{4\sin^2\phi + \sigma C_n} \quad (9)$$

$$b = \frac{\sigma C_t}{4\sin\phi\cos\phi - \sigma C_t} \quad (10)$$

R is the local radius and c is the chord length at each section.

2.1. Prandtl's Loss Factor correction

The above equations are only valid for rotors with infinite many blades. In order to correct for finite number of blades, Glauert [1] introduced Prandtl's tip-hub loss factor. In this method, a correction factor F, is introduced that corrects the loading.

Prandtl factor derived in Eqs. (6) and (7), the incremental thrust force and torque to be modified by:

$$dT = 4\pi BF\rho V_0^2 a(1-a) r dr \quad (11)$$

$$dM = 4\pi BF\rho V_0 \Omega b(1-a) r^3 dr \quad (12)$$

Where F, the tip-hub loss factor, is defined by

$$F = \begin{cases} F_{hub} & \text{if } r \le 0.8R \\ F_{tip} & \text{if } r > 0.8R \end{cases} \quad (13)$$

Where

$$F_{tip} = \frac{2}{\pi}\arccos\left[\exp\left(\frac{-B(R-r)}{2r\sin\phi}\right)\right]$$

$$F_{hub} = \frac{2}{\pi}\arccos\left[\exp\left(\frac{-B(r-r_{hub})}{2r\sin\phi}\right)\right]$$

After considering tip loss factor, Eqs. (9) and (10) should be changed to:

$$a = \frac{\sigma C_n}{4F\sin^2\phi + \sigma C_n} \quad (14)$$

$$b = \frac{\sigma C_t}{4F\sin\phi\cos\phi - \sigma C_t} \quad (15)$$

And thrust coefficient is defined by Eq. (16):

$$C_T = 4aF(1-a) \quad (16)$$

2.2. Spera's correction

When the axial induction factor becomes larger than 0.2, the original BEM theory becomes invalid, different empirical relations between the thrust coefficient C_T and a can be made, introduced a new correction for thrust coefficient was given by Eq. (17).

$$C_T = \begin{cases} 4aF(1-a) & \text{if } a \le a_c \\ 4F(a_c^2 + a(1-2a_c)) & \text{if } a > a_c \end{cases} \quad (17)$$

And axial induction factor is:

$$a = 1 + 0.5(K(1-2a_c)) - 0.5\sqrt{[K(1-2a_c)+2]^2 + 4(Ka_c^2 - 1)} \quad (18)$$

Where

$$K = \frac{4F\sin^2\phi}{\sigma C_n}$$

And a_c is approximately 0.2

The chord distribution as defined by:

$$c(r) = \frac{2\pi V_0 BEP}{BC_l \Omega}$$

BEP is the blade element parameter.

Then, the aerodynamic forces obtained from the momentum theory in Eqs. (1) and (2) are become as follow:

$$dT = 4\pi BF\rho V_0^2 a(1-a) r dr \quad (19)$$

$$dM = 4\pi BF\rho V_0 \Omega b(1-a) r^3 dr \quad (20)$$

The power output is defined by:

3. Proposed solution of axial induction factor

The angular induction factor is obtained by resolution Eq. (21).

$$b^2 + b(1 + \frac{X}{\lambda_r}) + \frac{1}{\lambda_r^2}(a(1 - X\lambda_r) - a^2) = 0 \quad (21)$$

Where X= C_l/C_d is the glid ratio

The second order equation, admit two solutions, the positive solution is expressed below:

$$b = -(1 + \frac{X}{\lambda_r}) + \sqrt{(1 + \frac{X}{\lambda_r})^2 - \frac{4}{\lambda_r^2}(a(1 - X\lambda_r) - a^2)} \quad (22)$$

The formulation of axial induction factor as follow:

$$\frac{b}{1-a}\frac{e^{\frac{s}{1-a}}}{\sqrt{1-(e^{\frac{s}{1-a}})^2}} - \frac{2}{\lambda_r^2}(1-a)(1-X\lambda_r-2a)\frac{1}{\sqrt{\left(1+\frac{X}{\lambda_r}\right)^2 + \frac{4}{\lambda_r^2}(a^2+(X\lambda_r-1)a)}}$$
$$+\left[1+\frac{X}{\lambda_r} - \sqrt{\left(1+\frac{X}{\lambda_r}\right)^2 + \frac{4}{\lambda_r^2}(a^2+(X\lambda_r-1)a)}\right]\cos^{-1}\left(e^{\frac{s}{1-a}}\right) = 0$$

4. Analysis results and discussions

The results are validated by comparing the results obtained from numerical code with the literature. The characteristics of rotor wind turbine: two blades, rotational velocity of 72 r/min, rotor diameter 10.06 m, chord and twist are varied, aerodynamic profile S809.

The equations cannot be solved directly, the solution is obtained by an iterative numerical approach in MATLAB code, and rapid convergence is obtained with a number of iterations.

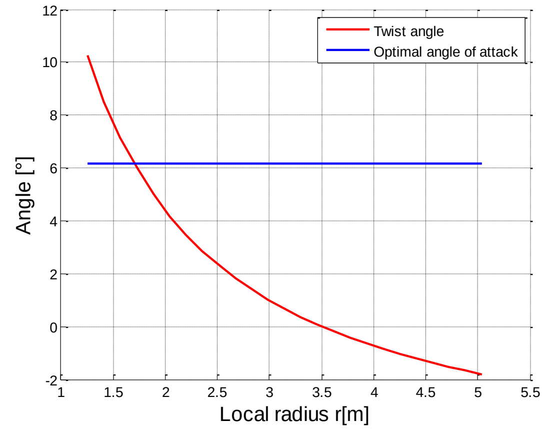

Fig.3: Twist and optimal angle of attack distribution

Figure 3 presented the twist angle distribution across the blade length varying from 10.25 degrees in root to nearly -1.8 degrees near tip of blade. The negative twist angle causes the elements of blade tip had a proper angle of attack in slow startup wind speeds.

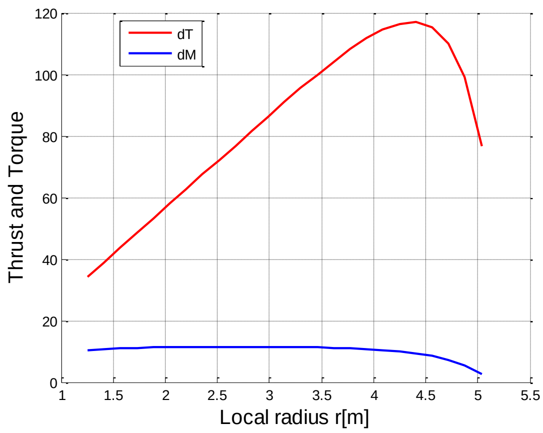

Fig.4: Thrust and torque distribution along the blade

The thrust and torque distribution are presented in Fig. (4), the thrust is varied linearly, but at the hub and tip of blade are curved by tip-hub loss factor effects. Especially in the tip region, the large over-prediction of loads is greatly reduced.

5. Conclusion

The optimum aerodynamic blade geometry was determined using iterative process for considering a new formulation of axial induction factor and tip loss correction factor. This result was compared with literature. In this study, the following remarks are addressed:

- The BEM analyzes for aerodynamic design of the optimal variable speed wind turbine blades are reexamined and improved.
- A new formulation for the axial induction factor, a, is introduced based on the present improved BEM method.

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