THERMAL CONVECTION CURRENTS ALONG AN INCLINED PLATE EMBEDDED IN SATURATED DARCY-BRINKMAN POROUS MEDIUM: EFFECTS OF THE CONTROLLING PARAMETERS.

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Abstract

An analysis is presented to investigate the combined effects of thermal radiation and internal heat generation on the boundary layer convection flow along an inclined plate embedded in saturated porous medium, using the Darcy-Brinkman formulation, taking into account the convective term. The governing boundary layer equations with the boundary conditions are first cast into a dimensionless form by a unique similarity transformation and the resulting coupled differential equations are then solved numerically by a computational program based on the fifth order Runge-Kutta scheme with shooting iteration technique. Also, the effects of physical parameters entering in the problem on the Nusselt number profiles have been computed and studied with help of graphics.

Mots clefs: Porous medium, Darcy-Brinkman model, Free convection, Internal heat generation, thermal radiation, Inclined plate.

1. Introduction

Convective flow through porous media is a branch of research undergoing rapid growth in fluid mechanics and heat transfer. This is quite natural because of its important applications in chemical engineering, soil physics, nuclear reactors, oil extraction, transport processes in aquifers and biological systems. The growing volume of work devoted to this area is amply documented in the recent excellent reviews by Vafai [3] and Nield and Bejan [4].

Many studies have appeared concerning the interaction of radiative flux with thermal convection flows. In view of this, M.A. Hossain and Pop [1] have analyzed the radiation effect on a free convection flow along an inclined plate in a porous space. Chamkha et al [2] analyzed the effect of radiation heat transfer on flow and thermal fields from an inclined plate embedded in a variable porosity porous medium due to solar radiation.. On the other hand, the radiation effects on natural convection in an inclined porous surface with internal heat generation have been studied by Ferdows et al.[5] These authors found that the skin friction coefficient and the rate of heat transfer increase with increasing the radiation parameter.

In a recent paper, Achemlal et al.[6] investigated the free convection over a vertical flat plate embedded in saturated a Darcy porous medium with a variable heat source and radiation flux. The present study is an extension of the work of Achemlal et al. [6] to Darcy-Brinkman model, taking into account the convective term.

2. Mathematical formulation and resolution method.

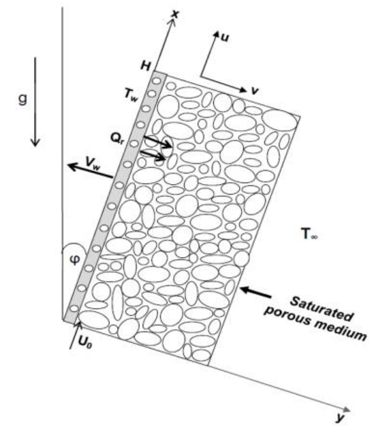

Fig.1: Physical configuration and coordinate system.

In this problem, the steady free convection heat transfer from an isothermal and permeable inclined plate embedded in a non Darcian porous medium saturated by a Newtonian fluid with internal heat generation and radiation effects is considered. The plate is inclined to the vertical direction by an angle φ. The x-axis is taken along the plate and y-axis is normal to the plate. The temperature of the plate is maintained at T_w, and the temperature of the fluid far away from the plate is T_∞. The Darcy-Brinkman model is used to describe the flow in porous medium, taking into account the convective term in momentum equation. It is assumed that the porous medium is isotropic and homogeneous, that all properties of the fluid and porous medium are constant, except the influence of density variation in the body force term. The convective fluid and the porous medium are in thermodynamic equilibrium anywhere. The flow is supposed two-dimensional, stationary and laminar for an incompressible fluid. The radiation heat flux in the x-direction is considered negligible in comparison to the y-direction. The physical model and coordinate system are shown in the figure 1.

Under the Boussinesq approximation and Taking into account the above assumptions. The governing equations describing the fluid flow can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right)\cos\varphi-\frac{\nu}{K'}u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}-\frac{Q_r}{\kappa}\right)+q''' \quad (3)$$

The associated boundary conditions are given by:

$$\begin{cases} y=0, x\geq 0, u=U_0, v=V_w(x), T=T_w \\ y\to\infty, x\geq 0, u=0, v=0, T\to T_\infty \end{cases} \quad (4)$$

where u and v are, respectively, the velocity components along x and y axes. The constants ν , K', α , g and β are respectively, kinematic viscosity, permeability of porous medium, thermal diffusivity, gravitational acceleration, and thermal expansion coefficient. U_0 and $V_w(x)=B.x^{-1/2}$ are respectively, the uniform velocity parallel to the vertical plate and the velocity of suction at the plate, where B is a constant.

The radiation flux on the basis of the Rosseland diffusion model for radiation heat transfer is expressed as :

$$Q_r=\frac{16\sigma^* T_\infty^3}{3kk^*}\frac{\partial T}{\partial y} \quad (5)$$

where σ^*, k are the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Here an exponential form is used for the internal heat generation and it is as follows:

$$q'''=\frac{U_0(T_w-T_\infty)}{2x}e^{-\eta} \quad (6)$$

Using (5) and (6), equation (3) takes the form:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}\left(1+\frac{16\sigma^* T_\infty^3}{3kk^*}\right)+\frac{U_0(T_w-T_\infty)}{2x}e^{-\eta} \quad (7)$$

The non-linearity of the model and the complexity of the phenomena encountered (boundary layer, instability, geometry of the porous medium ...) make difficult its resolution direct. The transformation of the PDE system, describing the problem studied in a set of non-linear differential equation becomes indispensable. So, we apply the following similarity transformations:

$$\eta=y\sqrt{\frac{U_0}{2\nu x}},\ \psi=(2\nu x U_0)^{1/2}f(\eta),\theta=\frac{T-T_\infty}{T_w-T_\infty} \quad (8)$$

where ψ is the stream function defined in the usual notation $u=\frac{\partial\psi}{\partial y}$, $v=-\frac{\partial\psi}{\partial x}$ and η the similarity variable.

After substitution and development, the governing equations (1), (2) and (7) gives rise to the following system of ordinary differential equations:

$$\begin{cases} f'''+ff''+Gr\theta\cos\varphi-Kf'=0 \\ (3R+4)\theta''+3R\Pr f\theta'+3R\Pr e^{-\eta}=0 \end{cases} \quad (9)$$

where $Gr=\frac{g\beta(T_w-T_\infty)2x}{U_0^2}$ is the Grashof number, $K=\frac{2\nu x}{K'U_0}$ is the permeability parameter, $\Pr=\frac{\nu}{\alpha}$ is the Prandtl number and $R=\frac{kk^*}{4\sigma^* T_\infty^3}$ is the radiation parameter.

The transformed boundary conditions for equations (4) are :

$$\begin{cases} \eta=0,\ f(0)=f_w,\ f'(0)=1,\theta(0)=1 \\ \eta\to\infty,\ f'(\infty)=0,\theta(\infty)=0 \end{cases} \quad (10)$$

where $f_w=-V_w\sqrt{\frac{2x}{\nu U_0}}$ is the suction parameter.

The system of the ordinary differential equations (9) with the boundary conditions (10) are solved numerically by using the fifth-order Runge-Kutta scheme associated with the shooting method.

3. Skin-friction coefficient and Nusselt number.

The physical quantities of most interest in such problems are the local skin friction coefficient C_f and local Nusselt number Nu, which are given, respectively, by the following equations.

$$C_f=\frac{2}{\sqrt{\mathrm{Re}}}f''(0),\ \ Nu=-\sqrt{\mathrm{Re}}\theta'(0) \quad (11)$$

where $\mathrm{Re}=\frac{U_0 x}{\nu}$ is the Reynolds number.

4. Results and discussions.

In this study, we have analyzed the effects of the radiation parameter R, the inclination angle φ , the suction parameter f_w, the Grashof number Gr, and the Prandtl number Pr on the free convection flow over an isothermal inclined plate embedded in saturated porous medium with an internal heat generation, using the Darcy-Brinkman model. We note that the fluid suction corresponds to $f_w > 0$.

Figure 2 depicts the heat transfer rate at the wall according to the inclination angle for an isothermal and permeable plate in a saturated porous medium with an internal heat generation for various values of the radiation parameter R, at Gr = 2, Pr = 0.71, f_w = 1 and K = 1. It is observed that, for all values of R, the heat transfer rate decreases with the increase in the inclination angle φ. Here, it clearly notable that the heat transfer rate is more important at the wall in the case where the plate is vertical $(\varphi = 0)$.

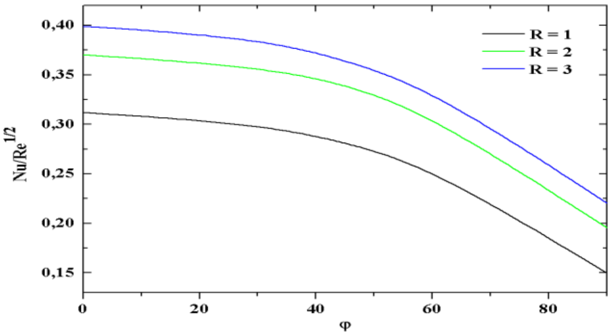

Fig.2: Nusselt number profiles versus φ for selected values of R.

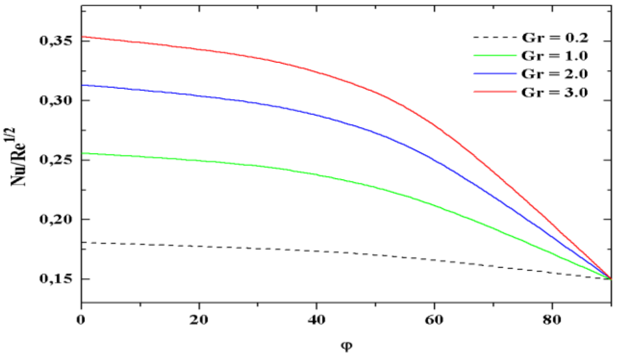

Fig.3: Nusselt number profiles versus φ for various values of Gr.

Figure 3, illustrates the local Nusselt number profiles according to the inclination angle φ at Pr = 0.71, R = 1, K = 1, and f_w = 1, for an isothermal plate embedded in a saturated porous medium with an internal heat generation. Here, it can be noted also that the heat transfer rate at the wall decreases for all values of Grashof number passing of φ = 0° to φ = 90°, and its observed clearly that the heat transfer rate becomes more important when the Grashof number augmented.

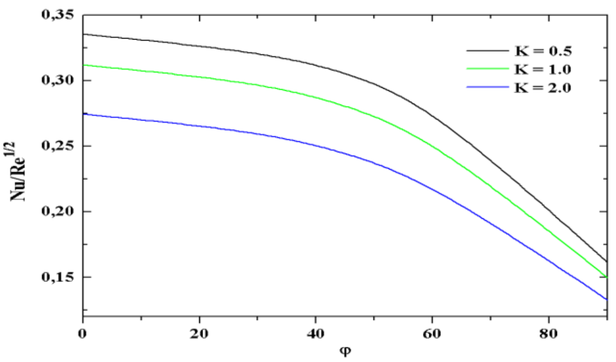

Fig.4: Nusselt number profiles versus φ for selected values of K.

In figure 4, the effects of the permeability parameter K on the local Nusselt number profiles according to the inclination angle φ at Gr = 2, Pr= 0.71, R=1 and f_w = 1, for an isothermal plate embedded in a saturated porous medium with an internal heat generation are shown. Here, it can be noted that the heat transfer rate at the surface plate decreases for the three values of permeability parameter K passing of φ = 0° to φ = 90°.

5. Conclusions

A numerical investigation has been made for a steady thermal convection currents around an isothermal and permeable inclined plate embedded in saturated porous medium, with a variable internal heat generation and radiation flux, using the Darcy-Brinkman model, and, taking into account the convective term. The effects of some physical parameters controlling the problem are investigated and presented graphically. The following main conclusions can be drawn from the present study:

- The wall heat transfer rate is optimal for the case where the plate is vertical.
- The radiation effect leads to reduce the heat transfer rate at the wall.
- The heat transfer rate at the wall becomes more important when the Grashof number increased
- With increasing of the permeability parameter and the inclination angle, the Nusselt number decrease.

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