Study the elastodynamic problem by Meshless Local Petrov-Galerkin Method using Laplace transform

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Abstract

The Meshless Local Petrov-Galerkin (MLPG) with Laplace transform are used for solving partial differential equation. The accuracy of the method present dependency on a number of parameters deriving from local weak form and different sub-domain sizes. In this paper, the meshless local Petrov-Galerkin (MLPG) formulation is proposed for forced vibration analyses. First the results are presented for different values of α_s. After, for fixed values of α_s and α_Q , and for 55 uniformly distributed nodes, results are presented for different time-step.

KEYWORD **: MLPG, weak form, Cantilever plates, elastodynamic, Laplace transform, moving least-squares approximation**

1. Introduction

The MLPG approach has been applied for solving 2D and 3D problems domain solutions by ; Atluri and al., [1,2], and for boundary integral equations, as the MLPG method by Sladek and al., [3,4].The MLPG method also applied to free and forced vibrations of a beam and a thin plate by[5] .These method is based on a local weak form and Moving Least Squares (MLS) approximation and proposed in this paper to extend the MLPG method to dynamic analysis and for solving the problem of a thin elastodynamic homogenous plate problem. In this study the Laplace transform is applied to eliminate the time variable, then, the local boundary integral equations are derived for Laplace transforms and the Stehfest inversion method is applied to obtain the time-dependent solutions. Moreover, both the contour and domain integrations can be easily carried out on rectangular sub-domains.

The paper is organized as follows: In section 2 the Basic equations of elastodynamics and their Laplace transforms are proposed. in section 3 The MLPG formulation including the local weak form in Laplace-transformed domain using weighted residual method locally from the dynamic partial differential equation. The numerical results and discussions for 2D problem example are given in section V. Finally, the paper ends with a conclusion.

2. Elastodynamic basic equations

The strong form of the initial boundary value problem for small displacement elastodynamics is as follows:

$$\sigma_{ij,j} + b_i = \rho \ddot{u}_i + c\dot{u}_i \quad \text{in } \Omega \quad i,j = 1,2 \qquad (1)$$

The Laplace-transform of a function f(x, t) is defined as:

$$\mathrm{L}\big(\mathrm{f}(\mathrm{x},\mathrm{t})\big) = \bar{f}(x,s) = \int_0^{+\infty} f\,(x,t)e^{-st}\,d\tau \qquad (2)$$

Then the Laplace-transforms of the basic Eq. (1) is given by :

$$\bar{\sigma}_{ij,j}(\mathrm{X},\mathrm{s}) - \mathrm{s}c\bar{\mathrm{u}}_i(\mathrm{X},\mathrm{s}) - \rho\mathrm{s}^2\bar{\mathrm{u}}_i(\mathrm{X},\mathrm{s}) = -\bar{\mathrm{F}}_i(\mathrm{X},\mathrm{s}) \qquad (3)$$

Where

$$\bar{\mathrm{F}}_i(\mathrm{X},\mathrm{s}) = c\bar{\mathrm{u}}_i(\mathrm{X},0) + \rho\mathrm{s}\bar{\mathrm{u}}_i(\mathrm{X},0) + \rho\dot{\bar{u}}_i(X,0) + \bar{b}_i(X,s) \qquad (4)$$

3. The MLPG weak formulation in Laplace-transformed

Instead of writing the global weak-form for the governing equations, the MLPG methods construct the weak form over local subdomains such as Ω_s**.** The local weak-form of the governing equation (1) and by considering:

$$u^h(X,t) = \sum_{i=1}^{n} \phi_i(\mathrm{X})\mathrm{u}_i(\mathrm{t}) = \boldsymbol{\Phi}^{\mathbf{T}}(\mathrm{X})\,\mathbf{U}_s(\mathrm{t}) \qquad (5)$$

Where $\boldsymbol{\Phi}^{\mathbf{T}}(\mathrm{X})$ is the vector of MLS shape functions corresponding *n* nodes in the support domain of the point X, and can be written as

$$\boldsymbol{\Phi}^{\mathbf{T}}(\mathrm{X}) = \{\varphi_1(\mathrm{X}) \quad \varphi_2(\mathrm{X}) \quad \dots \quad \varphi_n(\mathrm{X})\}_{(1\mathrm{xn})}$$
$$= \mathbf{p}^{\mathbf{T}}(\mathrm{X})\boldsymbol{A}^{-1}(X)\boldsymbol{B}(X) \qquad (6)$$

With Laplace transform:

$$\bar{\mathbf{u}}^{\mathbf{h}}(\mathrm{x},\mathrm{p}) = \sum_{i=1}^{n} \varphi_i(\mathrm{x})\bar{\mathbf{u}}_i(\mathrm{p}) = \boldsymbol{\Phi}^{\mathbf{T}}(\mathrm{x})\,\bar{\mathbf{U}}(\mathrm{p}) \qquad (7)$$

We obtain:

$$\int_{\Omega_s} \left[\bar{\sigma}_{ij,j}(\mathrm{X},\mathrm{s}) - (\mathrm{s}c + \rho\mathrm{s}^2)\bar{\mathrm{u}}_i(\mathrm{X},\mathrm{s}) + \bar{\mathrm{F}}_i(\mathrm{X},\mathrm{s})\right]\Theta_i(X)\,d\Omega = 0,$$
$$\mathrm{i\ et\ j} = 1,2$$

Where $\Theta_i(X)$ is a test function.

Applying the Gauss divergence theorem one can write:

$$\int_{\partial\Omega_s} \bar{\sigma}_{ij}(X,s) n_j(X)\, \theta_i(x)\, d\Gamma - \int_{\Omega_s} \bar{\sigma}_{ij}(X,s)\, \theta_{i,j} d\Omega$$
$$-q \int_{\Omega_s} \theta_i(X) d\Omega_s\, \bar{u}_i(X,s) = -\int_{\Omega_s} \bar{F}_i(X,s) \theta_i(X) d\Omega \qquad (8)$$

Where $q = sc + \rho s^2$

$\partial\Omega_s$ is the boundary of the local subdomain which consists of three parts, i.e. (see **fig.1**)

$\partial\Omega_s = \Gamma_{si} \cup \Gamma_{st} \cup \Gamma_{su}$

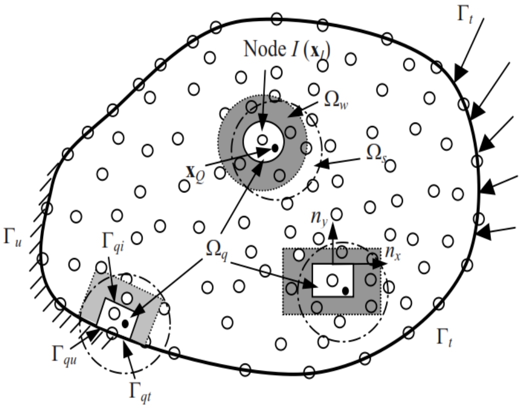

Figure 1 : The local sub-domains around point X_Q and boundaries

The local weak form (8) is leading to the following local boundary integral equation:

$$-\int_{\Gamma_{si}} \bar{t}_i(X,s)\theta_i(X)\, d\Gamma - \int_{L_{su}} \bar{t}_i(X,s)\theta_i(X) d\Gamma +$$
$$+\int_{\Omega_s} \bar{\sigma}_{ij}(X,s)\, \theta_{i,j} d\Omega + q\int_{\Omega_s} \theta_i(X) d\Omega_s\, \bar{u}_i(X,s)$$
$$= \int_{\Gamma_{st}} \tilde{\bar{t}}_i(X,s)\theta_i(X) d\Gamma + \int_{\Omega_s} \bar{F}_i(X,s)\theta_i(X) d\Omega \qquad (9)$$

The constitutive equation gives the relationship between the stress and the strain:

$$\bar{\boldsymbol{\sigma}}(X,s) = \mathbf{D} \sum_{k=1}^{n} \mathbf{B}_k(X)\bar{u}_k(s) \qquad (10)$$

The traction vectors $\bar{t}_i(x,p)$ at a boundary point $x \in \partial\Omega_s$:

$$\bar{\mathbf{t}}(X,s) = \mathbf{N}(X)\mathbf{D} \sum_{k=1}^{n} \mathbf{B}_k(X)\bar{u}_k(s) \qquad (11)$$

Substituting Eqs. (7), (10) and (11) into Eq. (9) we obtain the discretized LIEs:

$$-\sum_{k=1}^{n} \bar{u}_k(s) \int_{\Gamma_{si}} \mathbf{N}(X)\, \boldsymbol{\theta}_k(X)\, \mathbf{D}\mathbf{B}_k(X)\, d\Gamma$$
$$-\sum_{k=1}^{n} \bar{u}_k(s) \int_{\Gamma_{su}} \mathbf{N}(X)\, \boldsymbol{\theta}_k(X)\, \mathbf{D}\mathbf{B}_k(X)\, d\Gamma +$$
$$\sum_{k=1}^{n} \bar{u}_k(s) \int_{\Omega_s} \mathbf{W}_k \mathbf{D}\mathbf{B}_k(X)\, d\Omega + \qquad (11)$$
$$q\sum_{k=1}^{n} \bar{u}_k(s) \int_{\Omega_s} \varphi_k(X)\, \boldsymbol{\Theta}_k(X)\, d\Omega_s$$
$$= + \int_{\Gamma_{st}} \tilde{\bar{\mathbf{t}}}(X,s)\boldsymbol{\theta}(X) d\Gamma + \int_{\Omega_s} \bar{\mathbf{F}}(X,s)\boldsymbol{\theta}(X)\, d\Omega$$

The time dependent values of the transformed variables can be obtained by an inverse transform.

$$g_a(t) = \frac{ln2}{t} \sum_{i=1}^{N} v_i \bar{g}\left(\frac{ln2}{t} i\right)$$

Where

$$v_i = (-1)^{N/2+i} \sum_{k=[(i+1)/2]}^{\min(i, N/2)} \frac{k^{N/2}(2k)!}{(N/2-k)!\,k!\,(k-1)!\,(i-k)!\,(2k-i)}$$

4. Numerical results

In this section, we present a numerical study for elastodynamic 2-D problem of a cantilever rectangular homogeneous isotropic plate by using MLPG method, subjected to a dynamic force at the right end of the plate (See **Fig. 2).**

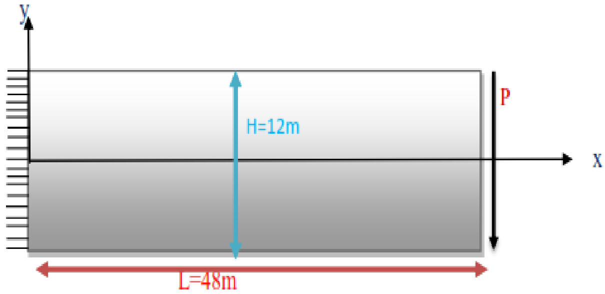

Figure 2 : Cantilever rectangular homogeneous isotropic plate subjected to a dynamic force at the right end of the plate.

We Consider a simple harmonic load $f(t) = \sin(wt)$ where w is the frequency of the dynamic load, and w=27 is used in this example. **Fig.2**.

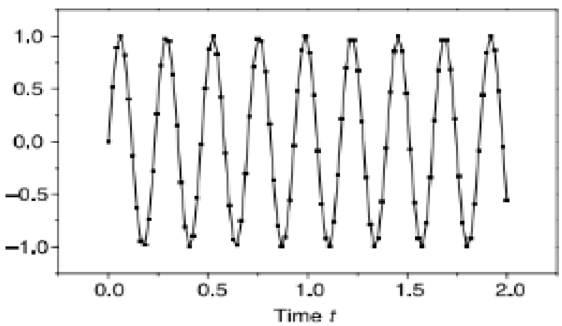

Figure 3 : A simple harmonic load g(t)

Fig.4 plots the displacement U_y at point A under the harmonic load for different values of $\alpha_s = 1.5, 2.0, 2.6$ *and* 3.0, where the time step is $\Delta t = 0.005$. It can be seen from Figure 5 that the size of the support domain influences on the results. when $\alpha_s = 3.0$, the results obtained by the present method is very good compared with the other authors that have used the Newmark method [3].

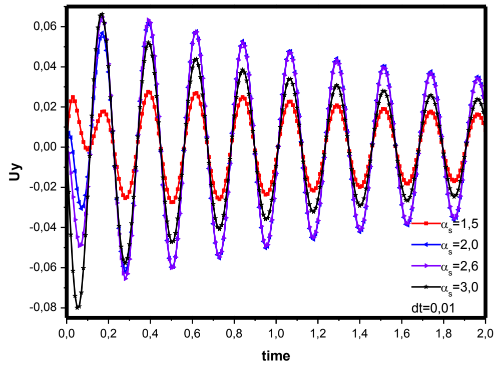

Figure 4 : Displacements U_y at the middle point at the free end of the plate excited by the time-step load (damping coefficient=0.4) for different values of α_s and with $\alpha_Q = 2.0$

In the following curves, $\alpha_s = 3.0$ is employed.

Results for different time steps are plotted in **Fig.5**. It can be found that when the time step Δt is less than 0.01, perfect results are obtained using the Laplace transform. It also can be found that when a time step Δt is larger than 0.01, the results are not reasonable any more.

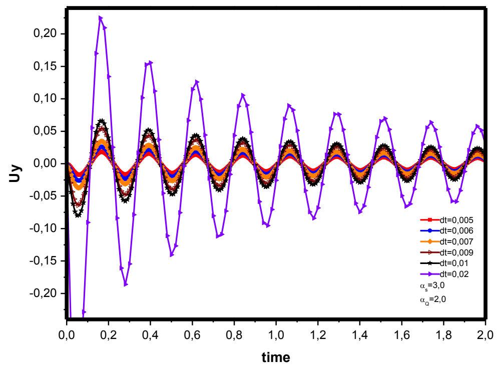

Figure 5 : The variation of displacement in the y direction at the point A with the time increment Δt

Conclusion

In this study the equation formulation based on meshless MLPG method and in the Laplace transform and time domain has been successfully implemented to solve 2-D elastodynamic problems for an isotropic solid, subjected to a dynamic force at the right end of a cantilever plate. We found that the amplitude of the vibration decreases with time because of the damping effects. It also can be found that when the time step Δt is less than 0.01, perfect results is obtained using the Laplace transform and when a time step Δt is larger than 0.01, the results are not reasonable any more.

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