

HYDRODYNAMIC PARAMETERS ESTIMATION IN A VERTICAL UNSATURATED SOIL COLUMN

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Abstract

In this study the hydrodynamic properties of an unsaturated soil were estimated simultaneously from a laboratory column infiltration experiment. Soil samples were collected from the area of Mnasra in north western of Morocco. As governing equations we used the Richards equation for variably saturated flow coupled with van Genuchten model. Inverse parameter estimation approach was used in which the unknown parameters were estimated with the least-square method through implementation of the Levenberg-Marquardt algorithm. Sensitivity coefficients were examined in order to determine the most meaningful measurements for identifying the unknown hydraulic parameters. Results obtained using the measured water content data during the unsaturated soil column experiment revealed the robustness of the proposed approach.

Keywords: *Hydraulic properties, Soil column, Experiments, Inverse estimation, Levenberg-Marquardt*

1. Introduction

Soil hydraulic parameters are essential inputs for simulating soil moisture and hydraulic conductivity. However, these parameters are difficult to obtain especially when the application is aimed at the regional scale [1,2]. The development of appropriate methods for the determination of the unsaturated flow properties of soils remains a challenging task for the soil science community and is needed for the many environmental engineering applications involving water and solute flow modeling. The knowledge of these hydraulic properties can be obtained by direct measurements in the laboratory or in situ.

Recently, one of the most popular experimental methods used to measure hydraulic properties of unsaturated soils in the laboratory is a one dimensional soil column experiments. Authors use typically three types of methodologies: one-step outflow experiments, multi-step outflow experiments and continuous flow experiments. These experiments were performed under transient or

steady conditions including wetting or drying processes. The hydraulic parameters of unsaturated soils were determined from discharge water velocity, evolution of soil suction and water content during the test.

Currently, numerical methods combined with an optimization code have become promising tools to solve hydraulic problems easily and effectively [3,4]. With these methods, the unknown parameters of hydraulic parameters are estimated by minimizing the difference between the predicted and measured measurements of soil pressure head, flow rate and water content. The application of the inverse modeling to one step outflow experiments was proposed early to estimate hydraulic properties using only cumulative outflow data [2,5]. Since soils are inherently heterogeneous and complex material, inversion analysis for hydraulic properties can lead to non unique solution. Therefore, to overcome non-uniqueness in inversion analysis, it has been recommended to include additional function $\theta(h)$, or tensiometer measurements in the objective function. Thus, combination of outflow, volumetric water content and suction over time could be used for the inversion of parameters for the soil water characteristic curve [3].

The aim of this study is to evaluate the applicability of an inverse parameter estimation method in a 1-D outflow experiment to determine the hydraulic properties of unsaturated soils. The saturation experiment was carried out on a vertical clay column to measure cumulative outflow, water content and suction of soil over time. Simultaneously, a finite difference code VZM and inversion code were developed for hydraulic parameter estimations using measured data of soil suction (from disc infiltrometer), water content (from PR2 probe) and the outflow rate from outflow experiments as input data. In addition, a comparison between predicted and measured hydraulic properties was also performed to determine the optimal inverse parameter estimation of unsaturated hydraulic properties of homogeneous soil taken from Mnasra region.

2. Material and methods

2.1 Unsaturated hydraulic properties

The one-dimensional vertical model in the unsaturated water flow equation is expressed as Richards's equation as follows:

$$\frac{\partial \theta(z,t)}{\partial t} = \frac{\partial}{\partial z} \left[K(h) \frac{\partial h(z,t)}{\partial z} - K(h) \right] \quad (1)$$

where K is the hydraulic conductivity [L/T], z is the soil depth (taken as positive downwards) [L], and t is the time [T]. Based on the initial and boundary conditions, Richards' equation was solved using the finite element method with an implicit scheme for time discretization. The discretized Richards' equation was resolved with a Fortran code VZM as described above.

To solve Richards' equation, the water retention function $\theta(h)$ and the hydraulic conductivity function $K(h)$ must be defined. We used the hydraulic model of the van Genuchten Mualem-type equations given by:

$$\theta(h) = \theta_r + (\theta_s - \theta_r) \left[1 + (|\alpha h|)^n \right]^{-m} \quad (2)$$

$$K(h) = k_s S_e^l \left(1 - (1 - S_e^{1/m})^m \right)^2 \quad (3)$$

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = (1 + (|\alpha h|)^n)^{-m} \quad (4)$$

in which θ_s is saturated water content; θ_r is residual water content of the soil; h the soil water pressure head; k_s is saturated hydraulic conductivity and S_e is the effective saturation.

2.2. Laboratory experiments

The physical problem considered here concern the flow infiltration through vertical unsaturated laboratory soil columns of length 1m, as shown in Figure 1. Values of the water content are measured instantaneously by PR2 Data logger. The PR2 Probes allows spatio-temporal measurements of water content at different depths in each column (Depths 10 and 30 cm). These measurements are used in the inverse approach for identifying the hydrodynamic parameters.

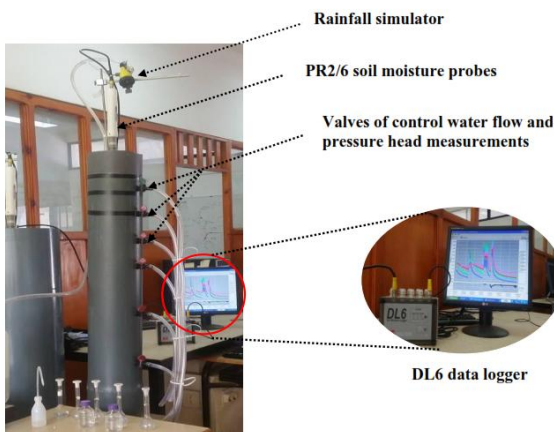


Figure 1. Experimental columns system used for drainage experiments.

2.2. Inverse optimization and multi-scale parameterization

The hydraulic parameters vector was estimated by minimizing an objective function $F(p)$, which was defined by the sum of the quadratic differences between the model-predicted water content and measured water content using the inverse optimization technique of the Levenberg-Marquardt method (Marquardt, 1963). $F(p)$ can be calculated with:

$$\Phi(p_k) = \sum_i^{n_z} \sum_j^{n_t} \sigma_i (\theta_i^j(h, p_k) - \hat{\theta}_i^j)^2 \quad (5)$$

where n_z is the number of measurements, n_t the number of time steps, θ_i the calculated values, $\sigma_i = 1/(\theta_{\min} - \theta_{\max})^2$ is the measurement weighting matrix. Differentiation of the objective function with respect to the parameter vector leads to a set of m nonlinear equations for the n unknown parameters, which has to be solved iteratively. Solving the inverse problem consists of choosing p_k such that measured and simulated values are in optimal agreement and the solution will be at the point where the cost function (Eq.5) has zero gradient.

3. Results and discussion

The aim of this study was to determine the soil hydraulic properties of Mnasra clay using the Levenberg-Marquardt algorithm from soil water content observation in laboratory soil column experiment.

The direct simulation is performed with the VZM code, using functional Van Genuchten-Mualem. The analysis of the evolution of the water content at different depths helps to understand the spatial and temporal aspects of water transfers. Single objective optimization approach was employed to estimate model parameters in equation (1), where only information on water content data was incorporated in the objective function to identify hydraulic parameters. The system response in the simulation and observation is shown in Figures 1 and 2. The solid lines are the water content rates simulated using the calibrated parameter set, it can be found that the measured and simulated values are in good arrangement.

Table 1. Results of estimated parameters

Parameter	Calibrated value by direct method	Estimated value by inverse method
θ_r (cm ³ cm ⁻³)	0.00	0.0010
θ_s (cm ³ cm ⁻³)	0.31	0.345
n (-)	2.36	2.23
α (cm ⁻¹)	0.366	0.320
k_s (cm s ⁻¹)	0.0083	0.0092

The results of the inversions performed with the Levenberg-Marquardt algorithm are summarized in

Table 1. The match obtained for the Van Genuchten model is illustrated in Figures 2 and 3 indicating that the data contain sufficient information for a definite identification of the conceptual model. Note that the estimated hydraulic parameters values strongly depend on the selected characteristic curves (Fig.4 and Fig.5).

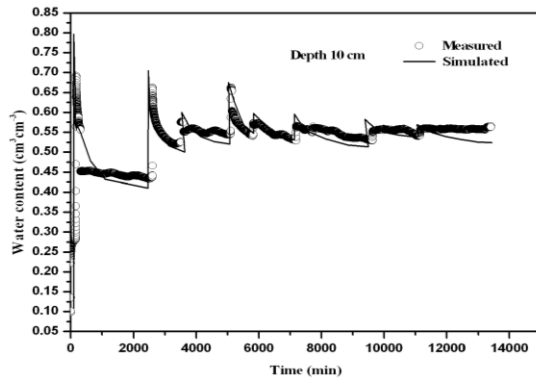


Figure 2. Measured and simulated water content at depth 10 cm.

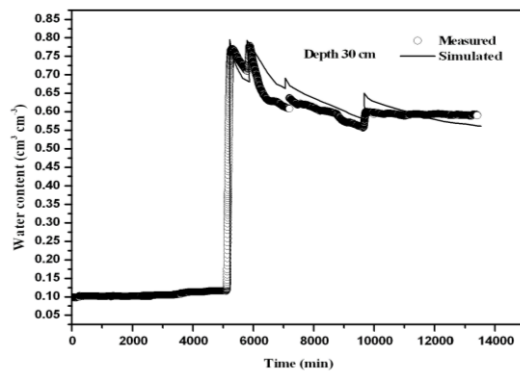


Figure 3. Measured and simulated water content at depth 30 cm.

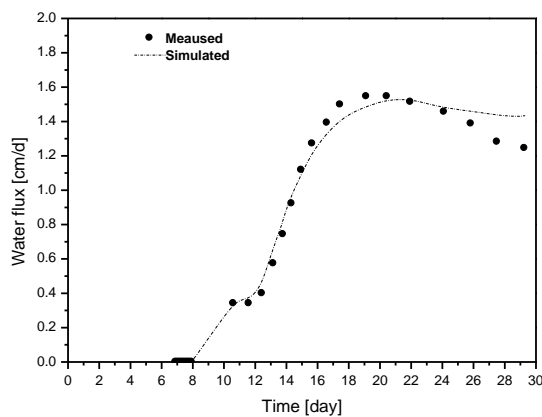


Figure 4. Bottom water flux curve

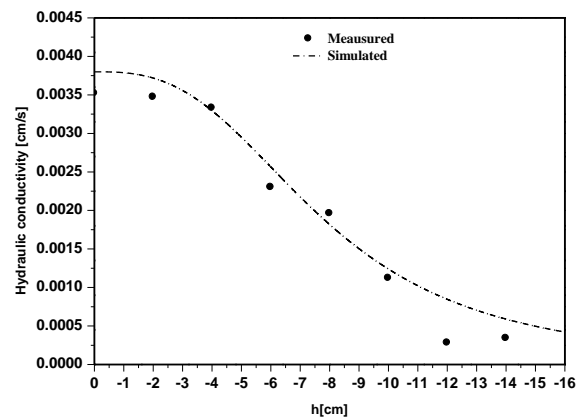


Figure 5. Hydraulic conductivity curve.

4. Conclusion

In this work, an inverse modeling is used to estimate the hydraulic properties of laboratory unsaturated soil column. The Levenberg–Marquardt algorithm with numerical direct solver of the one dimensional Richards' equation and experimental water content measurements series are used to minimize the objective function. The simulations of laboratory infiltration tests are confronted with the experimental data and the inverse model gives best values of hydraulic parameters.

Références

- [1] H. Qanza, A. Maslouhi, S. Abboudi. *Experience of Inverse Modeling for Estimating Hydraulic Parameters of Unsaturated Soils*. Russ. Meteorol. Hydrol. (2016) No. 11, pp. 68–80. DOI: 10.3103/S1068373916110054
- [2] M. Marinoni, F. Delay, Ackerer, P., Riva, M., & Guadagnini, A., 2016. Identification of groundwater flow parameters using reciprocal data from hydraulic interference tests. *Journal of Hydrology*, 539, 88-101.
- [3] J. Lai and L. Ren, "Estimation of effective hydraulic parameters in heterogeneous soils at field scale," *Geoderma*, 264, 28-41 (2016).
- [4] K. Tamoh and A. Maslouhi. Mesures in situ par infiltrométrie des propriétés hydrodynamiques des sols de Mnasra (Maroc). *Comptes Rendus Geoscience* (2004) 336(6), 535-545.
- [5] Levenberg, K. A method for the solution of certain problems in least-squares. *Quart. Appl. Math* (1944), 2, 164-168.
- [6] D.W. Marquardt. An algorithm for least-squares estimation of nonlinear parameters. *Journal of the society for Industrial and Applied Mathematics* (1963), 11(2), 431-441.