USING AN HYDRODYNAMIC ANALYTICAL MODEL FOR TIDAL DYNAMICS PREDICTIONS IN CONVERGENT ESTUARIES. APPLICATION TO THE SEBOU RIVER ESTUARY (MOROCCO)

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Abstract

Tidal dynamics in estuaries have long been the subject of intensive scientific interest. An adequate understanding of tidal wave propagation in estuaries is of great concern. In this study, a hydrodynamic analytical model for tidal wave propagation proposed by [1] and ameliorated later [2] has been applied to the Sebou river estuary. For given topography, friction, and tidal amplitude at the seaward boundary, the velocity amplitude, the wave celerity, the tidal damping, and the phase lag along the estuary axis are calculated.

Key-words: *Sebou estuary; tidal dynamics; hybrid analytical model; measurements; management.*

1.   Introduction

Ever since Barre de Saint-Venant published his equations for viscous flow in 1843, efforts have been made to solve the set of equations analytically. At first, out of need. Analytical solutions were the only option to turn the nonlinear partial differential equations into practical applications. In river systems, the equations could in general be simplified more easily than in tidal water bodies. Rivers tend to have more or less constant cross sections and in large rivers temporal gradients are slow. As a result, quasi steady state solutions or strongly linearized solutions appeared to work rather well in rivers, leading to analytical expressions for backwater curves, and the kinematic wave solution. In tidal water bodies, however, quasi steady state could not be assumed. Ever since Lorentz (1926) linearized the bottom friction term, simple solutions have been found to describe tidal dynamics with the linearized one-dimensional St. Venant equations. Since 1960s there exists a long tradition of analytical solutions for tidal dynamics in estuaries. These solutions usually made assumptions to simplify or linearise the nonlinear set of equations. Of these, most authors used perturbation analysis, where scaled equations are simplified by neglecting higher order terms, generally neglecting the advective acceleration term and linearising the friction term, higher-order terms, whereas [3] uses a simple harmonic solution without simplifying the equations. Others used a regression model to determine the relationship between river discharge and tide. Recently, [1] proposed a new analytical framework for understanding the tidal damping in estuaries. They concluded that the main differences between the examined models lie in the treatment of the friction term in the momentum equation. Furthermore, [1] presented a new 'hybrid' expression for tidal damping as a weighted average of the linearized and fully nonlinear friction term. Additionally, [2], including for the first time the effect of river discharge in a hybrid model that performs better. One-dimensional hybrid model can constitute the appropriate tool for quick-scan actions in a pre-phase of a project or for instructive purposes. Also, it is methodologically correct to start with the simplest description of the phenomena under study and to evaluate the limits of this approximation before investigating more complications.

The aims of this study it to investigate the applicability of hybrid model during 2015 year. A good fit was obtained between the computed and observed tidal dynamics evolution. The model constitute powerful tool for evaluating tidal wave propagation pattern in the Sebou river estuary.

2.   Mathematical Model

We consider a tidal channel with varying width and depth, a mostly rectangular cross section and lateral storage areas described by the storage width ratio r_s. The geometry of the idealized tidal channel is described in Figure.1, together with a simplified picture of the periodic oscillations of water level and velocity defining the phase lag. In a convergent estuary, where the main geometric parameters (tidally averaged cross-sectional area, width and depth) can be described by exponential functions. The tidal dynamics in an alluvial estuary can be described by the following set of one-dimensional dimensionless equations:

$$\frac{\partial U^*}{\partial t^*} + r_S \zeta \mu \lambda U^* \frac{\partial U^*}{\partial x^*} + \frac{\lambda}{\mu} \frac{\partial z^*}{\partial x^*} + \frac{\lambda}{\zeta \mu} \sigma^* ...$$
$$... + \mu \chi \frac{U^* |U^*|}{C^{*2} h^*} = 0 \qquad (1)$$

$$\frac{\partial z^*}{\partial t^*} + \zeta \mu \lambda U^* \frac{\partial z^*}{\partial x^*} + \lambda \mu h^* \frac{\partial U^*}{\partial x^*} - \mu \gamma h^* U^* = 0 \qquad (2)$$
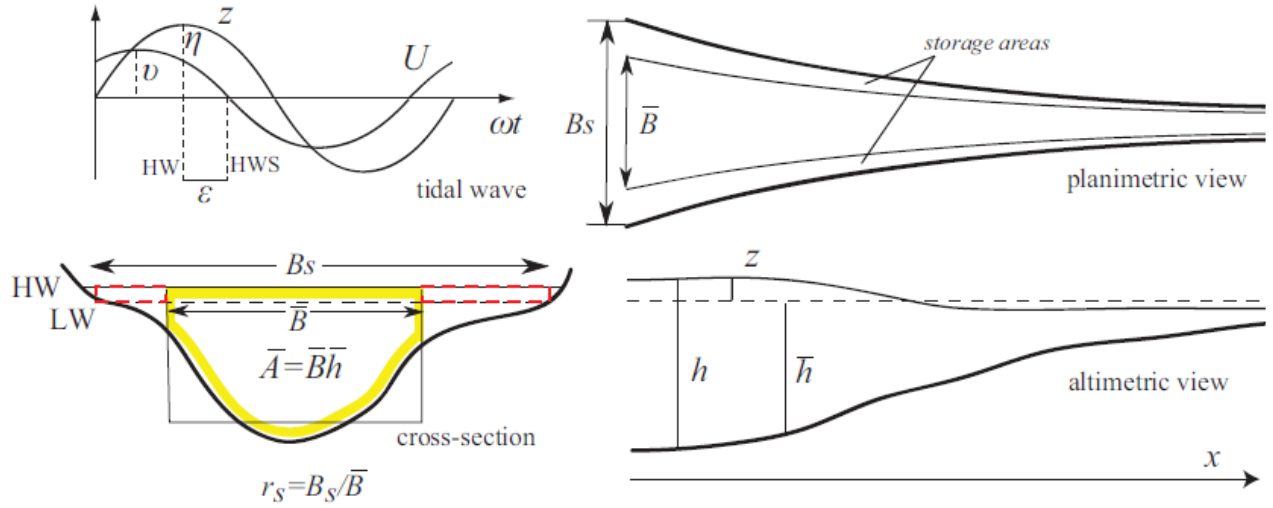
Figure .1: Sketch of the geometry of the idealized tidal channel and notation: tidal oscillations of water level z and velocity U and definition of the phase lag ε ; definition of the equivalent rectangular cross-section of width B, and of the total width B$_S$ including storage areas; planimetric view of the estuary with storage areas; lateral view showing instantaneous and tidally averaged depth [1], Figure modified from [3].

It is shown that the estuarine hydrodynamics are controlled by a few dimensionless parameters derived for the case of negligible river discharge: the estuary shape number γ , the friction number χ , the velocity number μ , the celerity number λ , the damping number for tidal amplitude δ and the phase lag ε.

The parameters γ and χ are the independent variables, while ε is defined as the phase difference between high water and high water slack, or between low water and low water slack. Due to the assumption of a simple harmonic solution for a simple harmonic wave: $\varepsilon = \pi / 2 - (\phi_H - \phi_U)$ where ϕ_H is the phase of water level and ϕ_U is the phase of tidal velocity), μ , λ , δ are the dependent variables. For further details on the scaling factors and the resulting dimensionless equations, readers can refer to [3]. These dimensionless variables are defined as:

$$\gamma = c_0 / \omega.a \qquad (3)$$

$$\chi = r_S .f .c_0 \zeta / \omega.\overline{h} \qquad (4)$$

$$\mu = v.\overline{h}/ r_S .\eta.c_0 \qquad (5)$$

$$\lambda = c_0 / c \quad where \quad c_0 = \sqrt{g\,\overline{h}/ r_S} \qquad (6)$$

$$\delta = c_0 d\eta / \eta.\omega.dx \qquad (7)$$

where $\omega = 2\pi/T$ is tidal frequency, c_0 is the classical tidal wave celerity of a frictionless progressive wave, c is the actual tidal wave celerity for average depth, η is the amplitude of water level, f is a dimensionless friction factor and R is the storage width ratio coefficient.

Utilizing these dimensionless parameters, [1], building on the previous work by [3], showed that the analytical solution of the tidal hydrodynamic equations can be obtained by solving a set of four implicit equations, i.e., the phase lag equation, the scaling equation, the damping equation and the celerity equation.

[1] proposed a hybrid expression for tidal damping by following the envelope method and considering the new friction term ($f = (g / K^2 \overline{h}^{\,1/3})/(1-(1.33.\eta / \overline{h})^2)$), leading to a revised damping equation accounting the effect of river discharge:

$$\delta = \frac{\mu^2}{1+\mu^2}.(\gamma - \frac{2}{3}\chi\mu^2\lambda^2 - \frac{8}{9\pi}\chi\mu\lambda) \qquad (8)$$

Equation (8), when combined with the phase lag; the scaling equation and the celerity equation forms a set of four implicit equations with four unknowns, which can be solved by an iterative numerical method.

Figure 1 shows the longitudinal computation of the tidal amplitude, the travel time (high water and low water), velocity amplitude and the dimensionless damping number in the case of Sebou estuary.

3. Results and discussion

Figure 2 shows that calculations by the model are in good correspondence with the observed data. The new model proposed by Cai et al, accounting the effect of river discharge is a powerful analytical instrument for conducting impact assessments induced by human interventions.
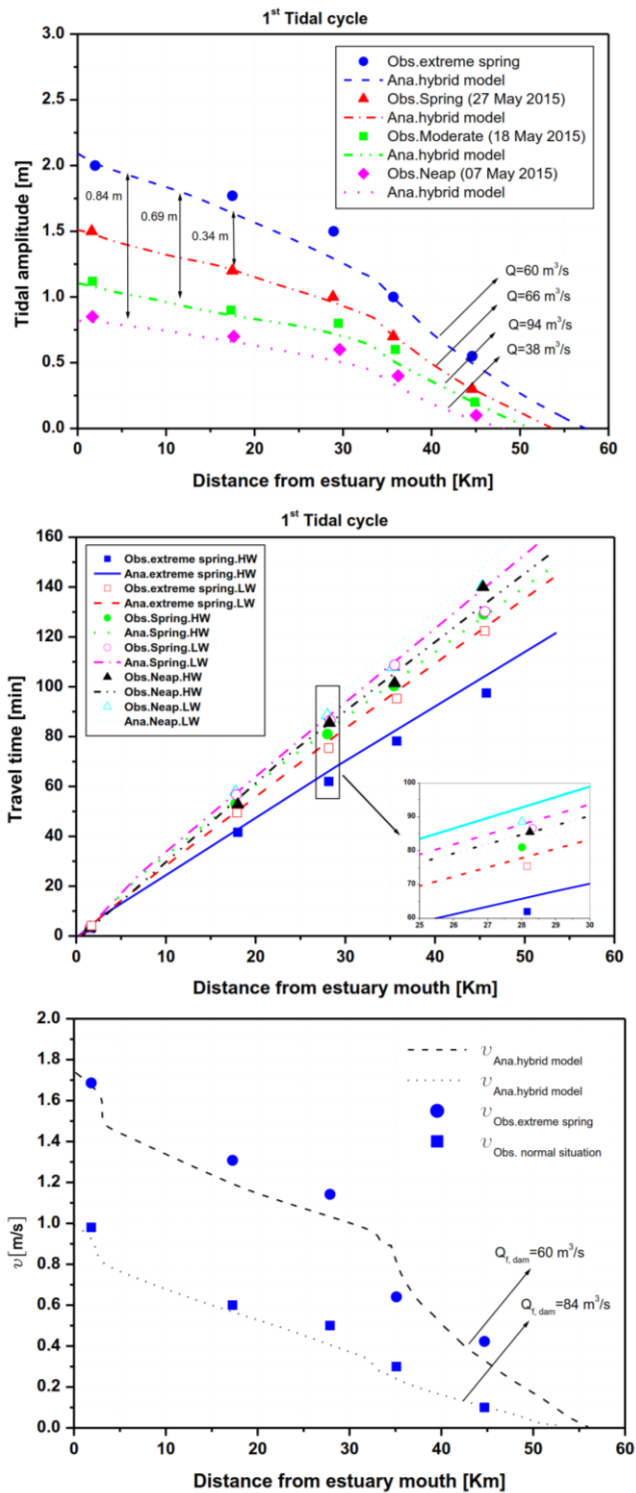
Figure 2: Comparison of analytically calculated tidal amplitude, velocity amplitude and travel time with measurements in the Sebou estuary.

4.   Conclusion

In this presentation, an analytical hydrodynamic model developed recently is applied in the Sebou river estuary. The new model proposed by Cai et al, is a powerful tool to predict river estuary hydrodynamic parameters like tidal amplitude, travel time and velocity amplitude.

In addition, it is worth noting that the results obtained using the analytical model, presented in this study, is actually used in combination with a salt intrusion model of the Sebou estuary [4, 5].

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