Taylor-SPH method for viscoplastic damage material

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Abstract

In this paper, we apply the Taylor-SPH (TSPH) meshfree method to solve the propagation of shock wave in 1D viscoplastic damage material. The numerical results show that the TSPH avoids the tensile instability, minimizes the numerical damping and dispersion and it performance to predict the damage of materials under dynamic conditions.

Key words: *Taylor-SPH; meshless; viscoplastic; damage*

1. Introduction

Finite Element Method (FEM) has been widely used to solve the PDEs in Mechanic and Engineering problems. Despite of it great success, FEM suffers from some difficulties when dealing with problems where extremely large deformations, moving boundaries, discontinuities or crack propagation are involved. To overcome these difficulties, various meshless methods were developed in the 90s. The first meshless method is the Smoothed Particle Hydrodynamics (SPH). It was originally introduced by Lucy [1] and Gingold and Monaghan [2] to model astrophysical problems. However original SPH suffers from some numerical difficulties: tensile instability and lack of consistency. Over the last decades, many corrections have been introduced to improve the consistency and the accuracy of the SPH solution. To eliminate the tensile instability, Dyka et al. [3] have proposed to insert additional points called stress points into SPH formulation. Nevertheless, the SPH method still presents some difficulties: numerical damping and dispersion when dealing with dynamics and shock wave propagation leading to poor accuracy of the solution.

In this work, we present a new meshless method Taylor-SPH [4-9] for solving the propagation of shock waves in solids. This meshfree method uses a two-steps time discretization algorithm by means of a Taylor series expansion and a corrected SPH method for the spatial discretization. In order to avoid the tensile instability, two different sets of particles are used. Both Lagrangian kernel and its gradient are corrected to satisfy the consistency conditions. The purpose of this paper is to show that Taylor-SPH avoids the tensile instability, minimizes the numerical damping and dispersion and it performance to predict the damage of materials in dynamic conditions. To avoid the loss of hyperbolicity of PDEs during the damage evolution, viscoplastic law coupled with damage model is adopted.

The paper is organized as follows. In Section 2, the stress-velocity mixed formulation for dynamic problems in elasto-viscoplastic damage material is presented. In Section 3, the PDEs are discretized using the Taylor-SPH meshfree method. To assess the performance of the proposed algorithm, some numerical examples are described in Section 4 using elasto-viscoplastic and damage materials.

2. Governing equations

2.1 Balance of momentum equations

For 1D bar of length L, the balance of momentum equation is given by

$$
\begin{aligned}
&\frac{\partial \sigma}{\partial x} = \rho \frac{\partial v}{\partial t} && in\ \left]0,\mathrm{L}\right[\, x\,(0,T) \\
&v(0,t) = v(t) && t \in (0,T) \qquad (1)\\
&v(L,t) = 0 && t \in (0,T) \\
&v(x,0) = v_o(x) && x \in [\,0,L]
\end{aligned}
$$

where σ is the stress, v is the velocity and ρ is the density.

2.2 Elasto-viscoplastic damage model

In this work, an elasto-viscoplastic law coupling with damage has been chosen to describe the material behaviour. The classical constitutive equation for isotropic damage model is given by [10, 11]:

$$
\sigma = (1-D)E\varepsilon^e \qquad (2)
$$

where D is the isotropic damage variable $(0 \le D \le 1)$. For the initial undamaged material $D=0$ and for completely failed material $D=1$. The damage evolution law used here is

$$
D(\kappa) = \begin{cases} 1-e^{-\alpha(\kappa-\kappa_o)} & \text{si } \kappa \ge \kappa_o \\ 0 & \kappa < \kappa_o \end{cases} \qquad (3)
$$

α is a model parameter, κ_o is the initial damage threshold and κ is the viscoplastic strain.

Differentiating eq. (2) with respect to time yields to

$$
\frac{\partial \sigma}{\partial t} = E(1-D)\left(\frac{\partial v}{\partial x} - \frac{\partial \varepsilon^{vp}}{\partial t}\right) - \dot{D}E\varepsilon^e \qquad (4)
$$

with $\qquad \dot{D} = \frac{\partial D}{\partial \kappa}\dot{\varepsilon}^{vp}$

As damage increases, the viscoplastic part of the model must incorporate the damage parameter

$$
\dot{\varepsilon}^{vp} = \gamma \left(\frac{f-(1-D)f_y}{(1-D)f_y} \right)^N \frac{\partial f}{\partial \sigma} \qquad (5)
$$

where $f = \sqrt{3J_2}$; being J_2 the second invariant of the deviatoric stress and f_y the yield stress. (γ,N) are the viscoplastic model parameters.

The system of first order PDEs for 1D elasto-viscoplastic damage model is given by

$$\frac{\partial v}{\partial t} - \frac{1}{\rho}\frac{\partial \sigma}{\partial x} = 0 \qquad (6)$$

$$\frac{\partial \sigma}{\partial t} - E(1-D)\frac{\partial v}{\partial x} = -E\left(1 - D + \varepsilon^e \frac{\partial D}{\partial \kappa}\right)\dot{\varepsilon}^{vp} \qquad (7)$$

In compact form, eqs. (6) and (7) can be written as

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A}\frac{\partial \mathbf{U}}{\partial x} = \mathbf{S} \qquad (8)$$

with $\mathbf{U} = \begin{pmatrix} \sigma \\ v \end{pmatrix}$; $\mathbf{A} = \begin{pmatrix} 0 & -E(1-D) \\ -1/\rho & 0 \end{pmatrix}$

and $\mathbf{S} = \begin{pmatrix} -E(1 - D + \varepsilon^e \dfrac{\partial D}{\partial \kappa})\dot{\varepsilon}^{vp} \\ 0 \end{pmatrix}$

It is important to notice that the eigenvalues of the matrix **A** are always real $\lambda = \pm c\sqrt{1-D}$.

Therefore the system of eqs. (8) remains hyperbolic and the problem still well-posed during the damage evolution. The use of the viscoplastic law regularizes the damage model and avoids the PDEs to become ill-posed.

3. Numerical discretization

3.1 Taylor-SPH Time discretization

First step: In order to obtain the time derivatives at the RHS of expression (8) at time t^n, the values of the unknowns at an intermediate time $t^{n+1/2}$ will be obtained first

$$\mathbf{U}^{n+1/2} = \mathbf{U}^n + \frac{\Delta t}{2}\left(\mathbf{S} - A\frac{\partial \mathbf{U}}{\partial x}\right)\Bigg|^n \qquad (9)$$

Second step: Substituting now the expressions obtained for the first and second order time derivatives in the Taylor series expansion, we obtain the values of the unknowns at time t^{n+1}:

$$\mathbf{U}^{n+1} = \mathbf{U}^n + \Delta t\left(\mathbf{S}^{n+1/2} + \mathbf{A}^n\frac{\partial \mathbf{U}}{\partial x}\bigg|^n - \mathbf{A}^{n+1/2}\frac{\partial \mathbf{U}}{\partial x}\bigg|^n - \mathbf{A}^n\frac{\partial \mathbf{U}}{\partial x}\bigg|^{n+1/2}\right) \qquad (10)$$

3.2 Taylor-SPH spatial discretization

We introduce an auxiliary set of particles called "virtual particles" interspersed among the real particles as it is shown in Fig.1.

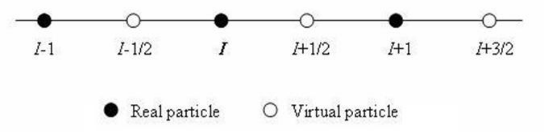

Fig.1. Real and virtual particles in 1D problem

First Step: By applying the corrected SPH spatial discretization to equation (9), we get the vector $\mathbf{U}_I$ at $t^{n+1/2}$:

$$\mathbf{U}_I^{n+1/2} = \mathbf{U}_I^n + \frac{\Delta t}{2}\left[\sum_{J=1}^{Nr}\frac{m_J}{\rho_J}\mathbf{S}_J^n\,\tilde{W}_{IJ} - \sum_{J=1}^{Nr}\frac{m_J}{\rho_J}A_I^n U_{IJ}^n\,\tilde{\nabla}W_{IJ}\right] \qquad (11)$$

where I = virtual particle; J = real particles

Second Step: Applying the corrected SPH spatial discretization to eq. (10), we obtain the vector $\mathbf{U}_I$ at t^{n+1}:

$$\mathbf{U}_I^{n+1} = \mathbf{U}_I^n + \Delta t\left[\sum_{J=1}^{Nv}\frac{m_J}{\rho_J}\mathbf{S}_J^{n+1/2}\,\tilde{W}_{IJ} + \sum_{J=1}^{Nv}\frac{m_J}{\rho_J}A_I^n U_{IJ}^n\,\tilde{\nabla}W_{IJ} - \sum_{J=1}^{Nv}\frac{m_J}{\rho_J}A_I^{n+1/2}U_{IJ}^n\,\tilde{\nabla}W_{IJ} - \sum_{J=1}^{Nv}\frac{m_J}{\rho_J}A_I^n U_{IJ}^{n+1/2}\,\tilde{\nabla}W_{IJ}\right] \qquad (12)$$

where I = real particles ; J = virtual particles

It is important to mention that with Taylor-SPH method, the PDEs are written in terms of stress and velocity and thus only essential boundary conditions must be considered.

4. Numerical examples

4.1. SPH tensile instability

The purpose of this Section is to show that the TSPH eliminates the traditional SPH tensile instability. The example consists of propagation of a shock wave into a 1D elastic bar of length L=0.1333 *m* (Fig.2). This example has been treated by Dyka et al. [8] to address the tensile instability. The applied boundary and initial conditions are:

- Boundary conditions: $v(x = L, t) = 0$
- Initial condition:

$$v(x, t=0) = v_o(x) = \begin{cases} -5\ \text{m/s} & for \quad x \le 3.3325 \text{ x } 10^{-2} m \\ 0 & for \quad x > 3.3325 \text{ x } 10^{-2} m \end{cases}$$

The bar is discretized using 41 real particles and 40 virtual particles. The calculation has been carry out with Courant number C=1. Fig. 3 shows the stress at point x=1.66 cm. The numerical solution using Taylor-SPH is free of oscillations and diffusion and the results are in complete agreement with the analytical solution.

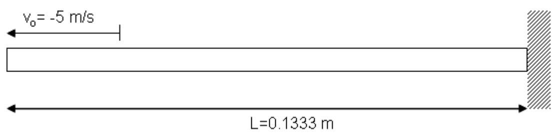

Fig. 2. 1D elastic bar

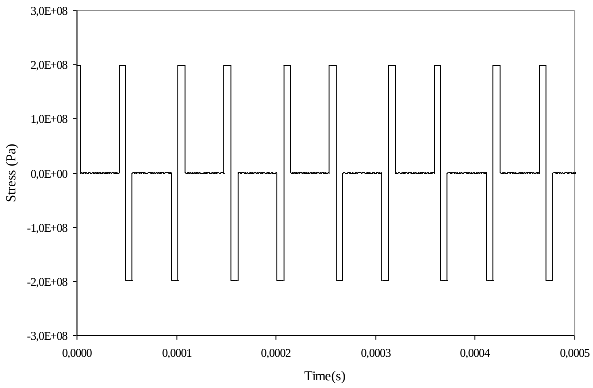

Fig. 3. Stress at point x=1.66 cm

4.2 Shock wave in viscoplastic damage bar

In this Section, the shock wave propagation in elasto-viscoplastic damage bar with a length of L=2m has been

studied. The parameters used in this example are: viscoplastic model parameters $(\gamma, N)=(5,1)$; yield stress $\sigma_o=10$ MPa and damage parameters $\alpha=5000$; $\kappa_o=10^{-4}$.
The bar is fixed at the right end $x=L$, and at the left end $x=0$ the velocity is imposed: $v(0,t)=v_o(t)=H(t_o-t)$ where H is the Heaviside function and $t_o=5\ 10^{-4}$s. The bar is discretized with 200 real particles. The bar behaves non-linearly and irreversible viscoplastic strains occur followed by damage of the material
In Fig. 4 is represented the evolution of damage variable D along the bar at different time. The material starts to be damaged at the fixed end $x = 2$ m just after reflection of the wave. Then damage propagates to locate on a reduced distance of the bar. Fig. 5 represents the stress-strain curves for two different refinements: 100 and 200 real particles. It can be observed that the results are similar and do not depend on the number of particles.

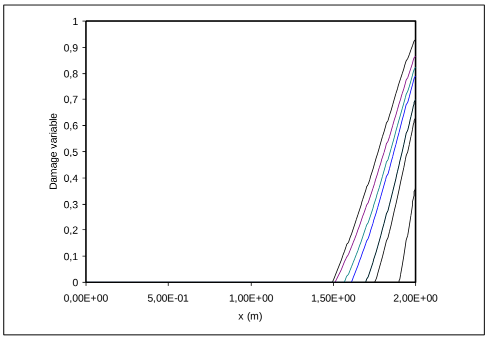

Fig. 4. Damage variable along the bar at different time

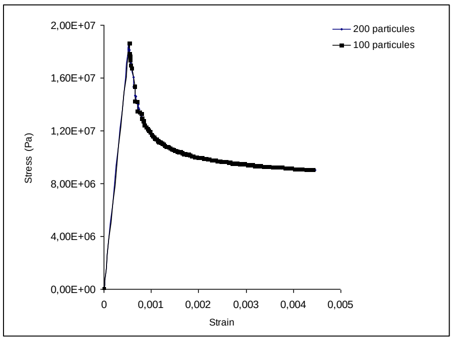

Fig. 5. Stress-strain curves for 100 and 200 real particles

5. Conclusions

In this paper, TSPH meshless method has been applied to shock waves propagating in 1D elasto-viscoplastic damage bar. The main advantages of this meshfree method are:

- It avoids the tensile instability
- It minimizes the numerical damping and dispersion when dealing with shock waves
- It is able to model localization phenomena for viscoplastic damage material without lose of hyperbolicity of PDEs.

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