Dynamic failure of viscoplastic materials using Taylor-SPH

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Abstract

In viscoplastic materials strain localization always occurs before failure. If the failure is produced by an impact, the problem is more complicated since usually shock waves are involved in the process. Taylor-SPH (TSPH) is a meshfree method suitable to solve the shock waves propagation in solids since it minimizes numerical dispersion and diffusion. It is able to capture shear bands accurately and it avoids the classical tensile instability. In this work, a set of numerical simulations has been carried out using TSPH. Results show that the TSPH method is an accurate tool to capture dynamic shear bands and therefore can be used to model failure of materials under dynamic conditions.

Keywords: Taylor-SPH; viscoplastic; failure; shear band

1. Introduction

The computation of shear bands in viscoplastic materials subjected to an impact is of paramount importance because of its importance in predicting failure of materials. The numerical predictions of failure could provide useful knowledge for engineering practice and design. Mesh based numerical methods such as the Finite Element Method has been successfully used to solve many problems in Solids Mechanics [1–3]. Nevertheless some difficulties arise when dealing with problems where shock wave propagation and localization of deformation are involved. To overcome these difficulties, meshfree methods have been actively developed recently. Among these methods, the Smoothed Particle Hydrodynamics (SPH) developed by Lucy [4] and Gingold and Monaghan [5]. Mabssout et al. [6–9] have proposed in previous works a new meshfree method, the Taylor-SPH (TSPH), to solve the shock propagation in solids. This new numerical method is suitable to solve the propagation of shock waves in linear and non linear materials and the localization of dynamic shear bands in viscoplastic softening materials. Thus it allows the prediction of dynamic failure in viscoplastic materials subjected to an impact. The TSPH method consists of applying first the time discretization by means of Taylor series expansion in two steps and a corrected SPH method for the spatial discretization. In order to avoid numerical instabilities, two different sets of particles are considered to perform the time discretization and a Lagrangian kernel is used. Both, Lagrangian kernel and its gradient, are corrected to satisfy the consistency conditions. In order to show that the TSPH method is an excellent tool to compute the dynamic shear bands and failure of materials, a set of numerical simulations has been carried out using TSPH. The paper is organized as follows. First, governing equations are given in Section 2. In Section 3, equations are discretized using the Taylor-SPH method. Numerical simulations are given in Section 4.

2. Governing equations

The governing equations are written in terms of stress and velocity. Neglecting the body forces, the governing equations can be written as

- *Balance of momentum equations*

$$\nabla \cdot \boldsymbol{\sigma} = \rho \frac{\partial \mathbf{v}}{\partial t} \qquad \text{in } \Omega \text{ x } [0,T] \tag{1}$$

$$\mathbf{v} = \bar{\mathbf{v}} \quad \text{on } \Gamma_v \text{ x}[0,\mathrm{T}] \;;\quad \boldsymbol{\sigma} = \bar{\boldsymbol{\sigma}} \quad \text{on } \Gamma_\sigma \text{ x}[0,\mathrm{T}]$$

$$\mathbf{v}\big|_{t=0} = \mathbf{v}_o \quad \text{in } \Omega$$

where $\boldsymbol{\sigma}$ is the stress , ρ the density and $\mathbf{v}$ is the velocity.

- *Constitutive equation*

The material behaviour can be described as

$$\frac{\partial \boldsymbol{\sigma}}{\partial t} = \mathbf{D}^e : \left(\frac{\partial \boldsymbol{\varepsilon}}{\partial t} - \frac{\partial \boldsymbol{\varepsilon}^{vp}}{\partial t} \right) \tag{2}$$

where $\mathbf{D}^e$ is the elastic constitutive matrix. The viscoplastic component is given by [11]:

$$\frac{\partial \boldsymbol{\varepsilon}^{vp}}{\partial t} = \gamma \left\langle \frac{F - F_o)}{F_o} \right\rangle^{\mathrm{N}} \frac{\partial F}{\partial \boldsymbol{\sigma}} \tag{3}$$

In equation (3), the symbol <.> represents the Macaulay brackets. γ is the fluidity parameter, $\mathbf{m}$ characterizes the direction of the plastic flow. N is a model parameter and F is a function describing a convex surface in the stress space. The value F_o characterizes the stress level below which no viscoplastic flow occurs.

In this work we will use the Modified Cam-Clay model yield surface. The yield surface of the modified Cam-Clay model has the form of an ellipsoid in the p-q plane and it is defined by [12]:

$$q^2 + M^2 p(p - p_c) = 0 \tag{4}$$

We choose the function F as

$$F = p + \left(\frac{q}{M} \right)^2 \frac{1}{p} \tag{5}$$

and $F_o = p_c$ (6)

where p is the hydrostatic pressure, q is the deviatoric stress, M is the slope of the failure line in the p-q plane and p_c is a hardening parameter characterizing the size of the ellipsoid. Each stress state has a loading surface defined by

$$p_y = p + \left(\frac{q}{M}\right)^2 \frac{1}{p} \tag{7}$$

In small strain analysis and for 2D plane stress problems, the constitutive and balance of momentum equations can be written as

$$\frac{\partial}{\partial t}\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\\ v_1\\ v_2\end{bmatrix} - \frac{\partial}{\partial x}\begin{bmatrix}D_{11}v_1\\ D_{12}v_1\\ D_{33}v_2\\ \frac{\sigma_{11}}{\rho}\\ \frac{\sigma_{12}}{\rho}\end{bmatrix} - \frac{\partial}{\partial y}\begin{bmatrix}D_{12}v_2\\ D_{22}v_2\\ D_{33}v_1\\ \frac{\sigma_{12}}{\rho}\\ \frac{\sigma_{22}}{\rho}\end{bmatrix} = \begin{bmatrix}-D_{11}\dot{\varepsilon}_{11}^{vp} - D_{12}\dot{\varepsilon}_{22}^{vp}\\ -D_{12}\dot{\varepsilon}_{11}^{vp} - D_{22}\dot{\varepsilon}_{22}^{vp}\\ -D_{33}\dot{\varepsilon}_{12}^{vp}\\ 0\\ 0\end{bmatrix} \tag{8}$$

where $\dot{\varepsilon}_{ij}$ is the derivative with respect to the time of the strain tensor components and D_{ij} is the component of the plane stress elastic matrix.

Therefore, (8) represents a system of first order hyperbolic PDEs where the viscoplastic strains affects only the RHS of the equations. Equations (8) can be alternatively written as

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{F} = \mathbf{S} \tag{9}$$

being $\mathbf{U}$ the vector of unknowns; $\mathbf{F}$ and $\mathbf{S}$ are the flux and source terms respectively.

3. Numerical discretization

The Taylor-SPH meshfree method [6–10] is used to solve the partial differential equations (9). It consists of applying first the time discretization and thereafter the spatial discretization using the corrected SPH method.

- *Time discretization*

Time discretization of equation (9) is carried out by means of a Taylor series expansion in time of $\mathbf{U}$ up to second order accuracy in two steps:

First step: The values unknowns at an intermediate time $t^{n+1/2}$ is obtained first

$$\mathbf{U}^{n+1/2} = \mathbf{U}^n + \frac{\Delta t}{2}(\mathbf{S} - \nabla.\mathbf{F})^n \tag{10}$$

Second step: The unknowns at time t^{n+1} is given by

$$\mathbf{U}^{n+1} = \mathbf{U}^n + \Delta t\,(\mathbf{S} - \nabla.\mathbf{F})^{n+1/2} \tag{11}$$

- *Spatial discretization*

The Taylor-SPH spatial discretization is carried out using two steps and two sets of particles [6-10].

First Step: Applying the corrected SPH spatial discretization to the first step of time discretization, (10), we obtain the values of the variable $\mathbf{U}$ at $t^{n+1/2}$

$$\mathbf{U}_{VP}^{n+1/2} = \mathbf{U}_{VP}^{n} + \frac{\Delta t}{2}\left[\sum_{J=1}^{Nr}\frac{m_J}{\rho_J}\mathbf{S}_J^n\,\tilde{W}_{IJ} - \sum_{J=1}^{Nr}\frac{m_J}{\rho_J}\mathbf{F}_{VP,J}^n\,\tilde{\nabla}W_{IJ}\right] \tag{12}$$

being J the "real" particles, such that $|\mathbf{x}_J - \mathbf{x}_{VP}| \le 2\,h_o$.

Second Step: Applying the corrected SPH spatial discretization to equation (11), we obtain the values of the variable $\mathbf{U}$ at t^{n+1}

$$\mathbf{U}_{RP}^{n+1} = \mathbf{U}_{RP}^{n} + \Delta t\left[\sum_{J=1}^{Nv}\frac{m_J}{\rho_J}\mathbf{S}_J^{n+1/2}\,\tilde{W}_{IJ} - \sum_{J=1}^{Nv}\frac{m_J}{\rho_J}\mathbf{F}_{RP,J}^{n+1/2}\,\tilde{\nabla}W_{IJ}\right] \tag{13}$$

where J are the "virtual" particles, such that $|\mathbf{x}_J - \mathbf{x}_{RP}| \le 2\,h_o$.

4. Numerical examples

It will be analysed in this section a two dimensional soil sample under plane stress conditions subjected to an impact on its upper face. The soil sample is given in Fig. 1 and it consists of a square of side 1 m, but for symmetry only one half will be considered in the analysis.

The applied boundary conditions are the following: on Γ_1 velocity has been set equal zero $v_x = v_y = 0$; on Γ_2 the symmetry results on $v_x = 0$ and $\sigma_{xy} = 0$; on Γ_3 is a stress free boundary, and therefore $\sigma_{xx} = 0$ and $\sigma_{xy} = 0$ and finally, velocity at Γ_4 is v_x=0 and $v_y = v(t)$ where $t_o = 5\ 10^{-2}$ s. The soil behaviour is modelled by the Perzyna's viscoplastic law with the modified Cam-Clay yield surface.

The density of the material is ρ = 2000 kg/m³, the elastic modulus is $E = 8\ 10^7\,Pa$ and Poisson's ratio is ν = 0.3. The parameters of Perzyna's model are γ = 20 s^{-1} and N = 1. The parameters of the modified Cam-Clay model are: $p_c = 6\ 10^5\,Pa$; $e = 1$; $\lambda = 1$; $\kappa = 0.08$ and $M = 1.2$. The domain is discretized using 861 real and 800 virtual particles, so that every 4 real particles form a square with a virtual particle placed in its centroid.

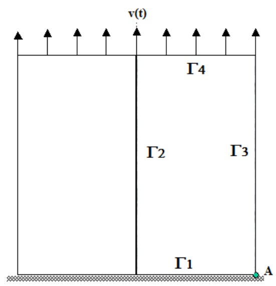

Fig. 1. 2D soil sample subjected to an impact

Fig. 2 shows the viscoplastic strain localization and the deformed configuration obtained using the TSPH method. This example shows that TSPH method is able to deal with shock waves propagating in 2D viscoplastic softening materials and it is an accurate tool to define the shear band before failure appears. The same example is solved using the following boundary conditions: $v_x = v_y = 0$ on Γ_1 ; $\sigma_{xx} = \sigma_{xy} = 0$ on Γ_2 and Γ_3 and finally v_x=0 and $v_y = v(t)$ on Γ_4. The soil sample is discretized using 3321 real particles (41 x 81 particles). Fig. 3 shows the accumulated viscoplastic strain localization for this case. Due to the boundary effect, two narrow shear bands starts at the bottom left and right corners. The accumulated viscoplastic strain localizes in the form of shear bands and propagates to the middle of the lateral side of the sample. Four shear bands appear, as illustrated in Fig. 3, which typically precedes failure of the material. This example demonstrates that the TSPH method is an accurate tool to capture shear bands location and inclination, and therefore, it can accurately predict where the solid will fail.

5. Conclusions

Taylor-SPH is a new meshfree method suitable to predict dynamic failure in viscoplastic materials. The problem of strain localization in viscoplastic materials have been investigated using the TSPH method. The obtained results show that the TSPH method avoids numerical instabilities of any kind and provides an accurate solution for the calculation of dynamic shear bands in softening viscoplastic materials. In conclusion, the TSPH method is an excellent tool to compute location and orientation of shear bands, information of paramount importance to predict the material failure.

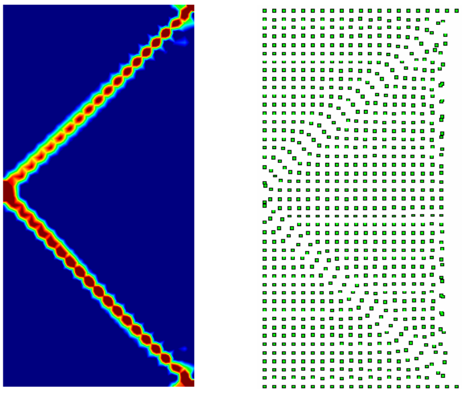

Fig. 2. Strain localization

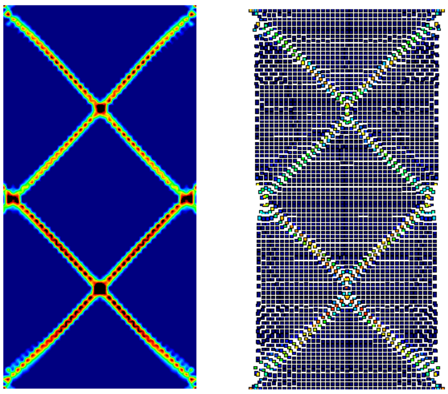

Fig. 3. Shear bands

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