EFFICIENT HIGHER ORDER SHEAR DEFORMATION THEORIES FOR FREE VIBRATION OF FUNCTIONALLY GRADED SANDWICH PLATES

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Abstract

In this paper, various four variable refined plate theories are presented to analyze vibration of functionally graded sandwich (FG) plates. By dividing the transverse displacement into bending and shear parts, the number of unknowns and governing equations for the present model is reduced, significantly facilitating engineering analysis. These theories account for parabolic, sinusoidal, hyperbolic, and exponential distributions of the transverse shear strains and satisfy the zero traction boundary conditions on the surfaces of the plate without using shear correction factors. Power law material properties and linear steady-state thermal loads are assumed to be graded along the thickness. Equations of motion are derived from Hamilton's principle. Analytical solutions for the free vibration analysis are obtained based on Fourier series that satisfy the boundary conditions (Navier's method). Non-dimensional results are validated with known results in the literature. Numerical investigation is conducted to show the effect of material composition and plate geometry, on the vibration characteristics. It can be concluded that the present theories are not only accurate but also simple in predicting the free vibration responses of FG plates.

Keywords: Functionally graded sandwich plate; Higher-order plate theory; Vibration;

1. Introduction

In recent decades, a new class of plates/shells made up of functionally graded materials (FGM), in which the material properties continuously vary through the thickness has become popular in various engineering applications. Because of the feature of continuously distributed material properties in FGM plates/shells, some drawbacks of conventional multilayered composite plates/shells resulting from the abrupt change of material properties at the interfaces between adjacent layers have been overcome, such as residual stress concentration, delamination, and matrix cracking. Consequently, this class of FGM plates/shells can provide more stable working performance than the conventional multilayered composite plates/shells usually achieve and has been successfully applied in various advanced industries. Therefore, developing theoretical methodologies and numerical modeling for the analysis of this class of FGM plates and shells has attracted considerable attention from researchers [1–7].

Recently, Mechab et al. [8] developed a new refined plate theory for FGM plates with only four unknown functions. How- ever, various higher-order shear deformation theories are developed using five unknown functions. Using the same methodology presented by Mechab et al. [8], in this paper, various efficient higher-order shear deformation theories are presented for free vibration analyses of functionally graded sandwich plates. The displacement fields of the present theories are chosen based on cubic (TSDT), sinusoidal (SSDT), hyperbolic (HSDT), and exponential (ESDT) variations in the in-plane displacement into the bending and shear parts and making further assumptions, the number of unknowns and equations of motion of the present theories is reduced and hence makes them simple to use. The novel feature of the theory is that it does not require shear correction factor, satisfying the shear-stress-free boundary conditions at top and bottom of the plate. Equations of motion are derived from Hamilton's principle. The accuracy of the present theories is verified by comparing the obtained results with the results obtained by the classical plate theory (CPT) and the first shear deformation theory (FSDT), and those obtained using higher order theories with five unknown functions.

2. PROBLEM FORMULATION:

Consider the case of a uniform thickness, rectangular FGM sandwich plate composed of three microscopically heterogeneous layers. The top and bottom faces of the plate are at $z = \pm h/2$, and the edges of the plate are parallel to axes x and y. The volume fraction of the FGMs is assumed to obey a power-law function along the thickness direction:

$$V^{(1)} = \left(\frac{z-h_1}{h_2-h_1}\right)^p, \quad z \in [h_1, h_2] \qquad \text{(1a)}$$

$$V^{(2)} = 1, \quad z \in [h_2, h_3] \qquad \text{(1b)}$$

$$V^{(3)} = \left(\frac{h-h_4}{h_3-h_4}\right)^p, \quad z \in [h_3, h_4] \qquad \text{(1c)}$$

The effective material properties like Young's modulus E, Poisson's ratio ν, and thermal expansion coefficient α then can be expressed by the rule of mixture [9–11] as

$$^{(n)}(z) = P_2 + (P_1 - P_2)\, V^{(n)} \qquad (2)$$

2.1. Governing equations

Using Hamilton's energy principle derives the equation of motion of the FG plate:

$$\delta \int_{t2}^{t1} (U - V - T)\, dt = 0 \qquad (3)$$

Where U is the strain energy, T is the kinetic energy of the FG plate, and V is the work of external forces.

3. NUMERICAL RESULTS:

Navier solutions for free vibration analysis of FG sandwich plates are presented by solving the eigen value equations. The FG plate is taken to be made of aluminum and alumina with the following material properties:

-Métal(Aluminum,Al) :

$$E_m = 70 \times 10^9\, N/m^2; \nu = 0{,}3; \rho_M = 2702 kg/m^3$$

Céramique(alumina,Al_2O_3) :

$$E_c = 380 \times 10^9\, N/m^2; \nu = 0{,}3; \rho_M = 3800 kg/m^3$$

The results of the power-law FGM sandwich plates with five material distributions are compared in Table 1 with the results based on CPT, FSDT. Table 1 shows a good agreement by comparisons of FGM plates of five different volume fraction indices with other higher theories.

It is seen from fig.1 that the increase in the power law index p produces a reduction in the fundamental frequency values. The results are the maximum for the ceramic plates and the minimum for the metal plates.

4. CONCLUSION:

In this paper, new shear deformation theories has been presented for thick functionally graded sandwich plates. The theories have the following features:

- It is a displacement-based theory that includes the transverse shear effects.
- The number of unknown functions involved in the theory is only four.
- The theory is variationally consistent.
- Transverse shear stress satisfies zero shear stress boundary conditions on top and bottom surfaces of the beam perfectly.
- The theory obviates the need of shear correction factor.

TABLE 1: comparisons of natural fundamental frequency parameters ϖ of simply supported square power-law FGM plates with other theories $(h/b = 0.1)$

p	theory	ϖ		
		1-0-1	2-1-2	1-1-1
0	CPT	1.87359	1.87359	1.8735
	FSDT	1.82442	1.82442	1.8244
	TSDT	1.82442	1.82442	1.8244
	SSDT	1.82452	1.82452	1.8245
10	CPT	0.94321	0.95244	1.0052
	FSDT	0.92508	0.93962	0.9925
	TSDT	0.92839	0.94297	0.9955
	SSDT	0.92875	0.94332	0.9951

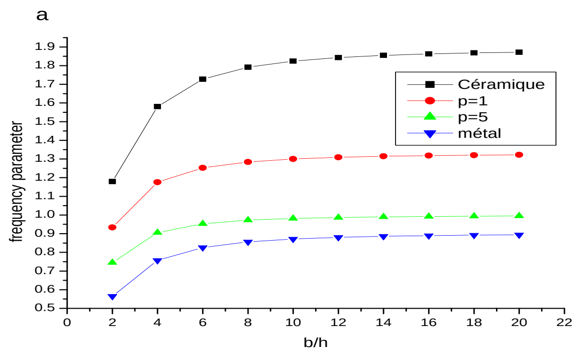

FIGURE 1 : fundamental frequency ϖ as a function of side-to-thickness ratio (b/h) of symmetric square fgm sandwich plates for various values of p. (a) the (2–1–2) FGM sandwich plate.

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