Effect of couple stress on flexible finite porous journal bearing

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Abstract

Based on the micro-continuum Stokes [1] theory, this paper investigates theoretically the rheological effects of couple stress fluids on the lubrication performances of flexible finite porous journal bearings. A modified Reynolds equation is derived in the fluid film. The flow in the porous elastic bearing is described by Darcy's law. These equations are discretized by finite differences method, coupled using a sequential algorithm and solved iteratively by Gauss-Seidel over-relaxation method. The numerical results of the present simulations show that all these effects have a significant influence on journal bearings performances.

Keywords: *Finite journal bearing; Non-Newtonian lubricants; Porous elastic bearing; Darcy model.*

1. Introduction

Porous journal bearings are made of a porous bush filled with lubricating oil, so that bearing requires no further lubrication during the whole life of the machine. They are widely used in home appliances, small motors, instruments and construction equipments because of their low cost and good bearing qualities. The analytical study of porous bearings was first made by Morgan *et al.* [2]. Elsharkawy *et al.* [3] the case of finite porous journal bearings lubricated with Newtonian fluid. However, in many lubrication problems lubricating oils contain suspended solid particles, the Newtonian fluid constitutive approximation is thus not a satisfactory engineering approach. Hence, the use of non-Newtonian fluids as lubricants has gained importance in the modern industry. Bujurke *et al.* [4] analyzed the performance characteristics of a court porous journal bearing lubricated with couple stress fluid ignoring the flexibility of the bearing. Heretofore, the effect of couple stress on the lubrication characteristics of flexible finite porous journal bearings have not been analyzed. The main intent of this paper is concerned with the effects of couple stress based on the Stokes micro continuum theory, on the lubrication performance of flexible finite porous journal bearings.

2. Governing equations

The journal rotates with a constant angular velocity about its axis. The porous elastic bearing is press-fitted in a rigid housing (Fig. 1).

In the fluid film using the usual assumption of hydrodynamic lubrication, the continuity and motion equations are written in Cartesian coordinates in the case of an incompressible, laminar, isothermal flow for couple stress [1] fluid in the absence of body forces and body couples:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{1}{\mu}\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2}-l^2\frac{\partial^4 u}{\partial y^4} \tag{2}$$

$$\frac{\partial p}{\partial y}=0 \tag{3}$$

$$\frac{1}{\mu}\frac{\partial p}{\partial z}=\frac{\partial^2 w}{\partial y^2}-l^2\frac{\partial^4 w}{\partial y^4} \tag{4}$$

where u, v, w, μ and p are respectively components of the lubricant in the x, y, and z directions, the dynamic viscosity and the pressure in the film. $l=\sqrt{\eta/\mu}$ is the couple stress parameter, where η represents a material constant responsible for the couple stress fluid property.

The porous elastic bearing is considered homogeneous and isotropic. Suspended particles in the fluid film are assumed sufficiently large to not penetrate into the bearing. This later is then saturated by an incompressible Newtonian fluid having the same dynamic viscosity as in the fluid film. The flow is described by Darcy model in cylindrical coordinates:

$$\frac{\partial p^*}{\partial r}=-\frac{\mu}{k}v^* \tag{5}$$

$$\frac{1}{r}\frac{\partial p^*}{\partial \theta}=-\frac{\mu}{k}u^* \tag{6}$$

$$\frac{\partial p^*}{\partial z}=-\frac{\mu}{k}w^* \tag{7}$$

Due to continuity of fluid in the porous elastic bearing, p^* satisfies the Laplace equation:

$$\frac{\partial^2 p^*}{\partial r^2}+\frac{1}{r}\frac{\partial p^*}{\partial r}+\frac{1}{r}\frac{\partial^2 p^*}{\partial \theta^2}+\frac{\partial^2 p^*}{\partial z^2}=0 \tag{8}$$

where p^*, k, v^*, u^* and w^* are respectively pressure, permeability, velocity components in r, θ and z directions, respectively, in the porous elastic bearing.

The boundary conditions of the velocity are: the no-slip condition $u=U$, $v=V$, $w=0$ at the journal surface (y=h+H), the continuity of the velocity field $u=u^*$, $v=v^*$, $w=w^*$ at the film-porous bearing interface

($y=H$), the impermeability condition $v^*=0$ for the lubricant at the external surface of the porous bearing ($r=0$). Due to the absence of local fluid particles rotation at the interface between the fluid and solid boundaries at ($y=H$ and $y=h+H$), the no-couple stress boundaries are used.

Integrating the continuity equation (1) with respect to y from H to $H+h$ and using analytical solutions u and w of motions equations (2-4), accounting for the specified boundary conditions, the modified Reynolds equation in the fluid film is then derived:

$$\frac{\partial}{\partial x}\left(f(h,l)\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(f(h,l)\frac{\partial p}{\partial z}\right)=-\mu v^*_{y=H}+\mu w^*_{y=H}\frac{\partial H}{\partial z}$$
$$+\mu\frac{\partial}{\partial z}\left(h w^*_{y=H}\right)+\mu\left(u^*_{y=H}-U\right)\frac{\partial H}{\partial x}$$
$$+\mu\frac{\partial}{\partial x}\left(\frac{h}{2}\left(U+u^*_{y=H}\right)\right) \qquad (9)$$

where $f(h,l)=\dfrac{h^3}{12}-hl^2+2l^3\tanh\left(\dfrac{h}{2l}\right)$

The film thickness h and the porous thickness H are:

$$h(x,z)=C(1+\varepsilon\cos\theta)+C_0,\quad H(x,z)=H_0-C_0 \qquad (10)$$

where C is the radial clearance, ε the eccentricity, H_0 the underformed thickness of the porous elastic bearing and C_0 is the compliance coefficient [5] given by:

$$C_0=\frac{H_0(1+\nu)(1-2\nu)}{E(1-\nu)}\,p(x,z) \qquad (11)$$

where E and ν are respectively the Young modulus and the Poisson ratio.

The associated boundaries conditions are: $p=p^*=0$ at the end of the porous bearing. In the circumferential direction the Reynolds boundary conditions [6] are adopted in the fluid film, they respect the continuity of flow and assume that for an unknown abscess x_θ, the pressure and the pressure gradient are:

$$p(x=x_\theta,z)=\frac{\partial p}{\partial\theta}(x=x_\theta,z)=\frac{\partial p}{\partial z}(x=x_\theta,z)=0$$

The impermeability condition at the external surface of the bearing is: $\dfrac{\partial p^*}{\partial r}(r=R+H_0,\theta,z)=0$.

The pressure is assumed to be continuous $p(x,y=H,z)=p^*(r=R,\theta,z)$ on the porous interface.

3. Numerical resolution

The set of the partial differential equations (8-9) cannot be solved analytically. To obtain approximate solutions, we use the finite differences method which approximates the differential equations by a system of algebraic equations. In order to easily perform a finite differences discretization, the physical domain is thus transformed into a rectangular domain and non-dimensional variables are introduced.

4. Results and discussions

According to the Stokes theory, the new material constant η is responsible for the couple stress property. Since the dimension of $l=(\eta/\mu)^{1/2}$ is length, this length may be identified as the characteristic length of suspended additives in a Newtonian lubricant. With the aid of the dimensionless parameter $\bar{l}=l/C$, this couple stress parameter characterizes the effect of couple stress on the bearing characteristics. The porous interface deformation is obtained using the thin elastic layer approach. The permeability parameter is defined by $\varphi=k/C^2$. The results are obtained for eccentricity ratios ε ranging from 0.1 to 0.8, a journal radius to bearing length R/L of 0.5, couple stress parameters $\bar{l}$ ranging from 0 to 0.4 and permeability parameters of 0, 5×10^{-6} and 5×10^{-5}.

Fig. 2 shows the dimensionless load-carrying capacity versus the eccentricities ε for different couple stress parameters $\bar{l}$. As shown, the couple stress effects increase the load-carrying capacity at a given eccentricity ratio ε. A larger increment is obtained with increasing values of ε and $\bar{l}$. It can be seen from Fig. 3 that as the permeability parameters φ increase the load-carrying capacity decreases and this decreases is more pronounced for high eccentricities ε. Fig. 4 depicts the dimensionless load-carrying capacity for different eccentricity ratios ε for both rigid and elastic porous bearing for a couple stress parameter $(\bar{l}=0.2)$ and a permeability parameter $(\varphi=5\times10^{-6})$, it is observed that the load-carrying capacity for rigid finite porous journal bearing is greater than that for elastic one, especially at high eccentricity ($\varepsilon>0.6$). This confirms the previous predictions by Nabhani *et al.* [7].

5. Conclusion

A modified Reynolds equation is derived using the Stokes constitutive equations. The flow is described by Darcy model in the porous bearing. Those equations are coupled at the elastic porous interface by the continuity of pressure and velocity. From the results presented in this paper, the following conclusions can be drawn:

The influence of couple stresses on the dimensionless load-carrying capacity is significant and not negligible. Compared with the Newtonian-lubricant case, the effects of the couple stresses provide an enhancement in the load-carrying capacity. The dimensionless permeability parameter φ has significant effects on the dimensionless load-carrying capacity especially at higher eccentricity ratios. As φ increases, the load carrying capacity decreases and it is shown that the elastic deformation of the porous bearing should be included in calculating the performance characteristics. The elasticity effect reduces the increase in the load-carrying capacity.

References

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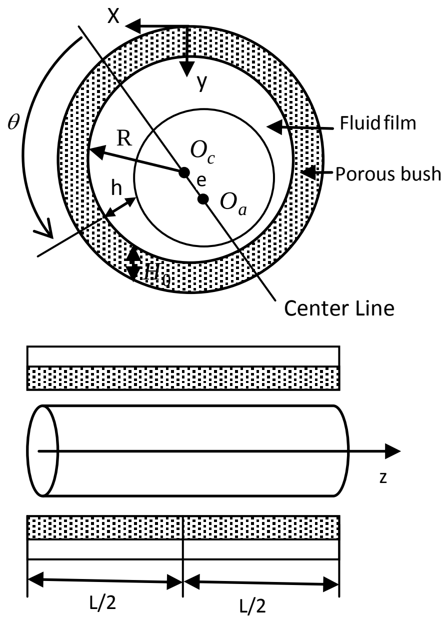

Fig. 1. Porous journal bearing configuration.

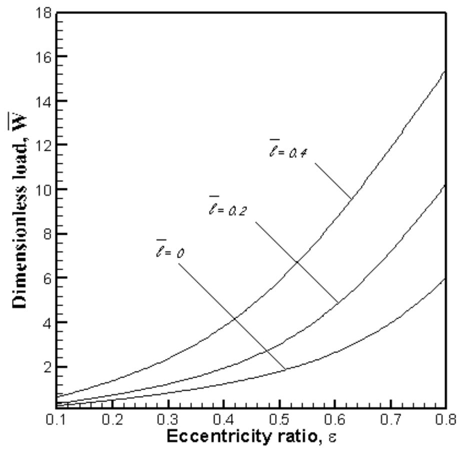

Fig. 2. Dimensionless load-carrying capacity $\overline{W}$ for various values of couple stress parameter $\bar{l}$ at $\varphi = 5\times10^{-6}$ and $C_0 = 0.01$.

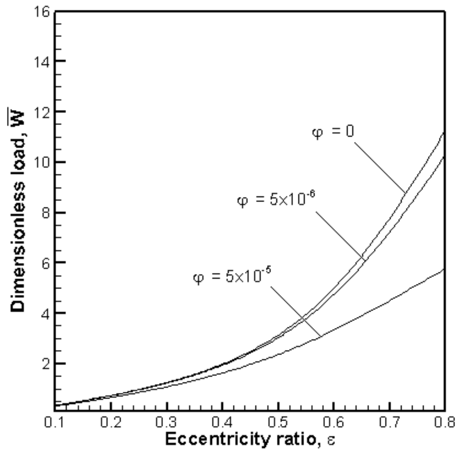

Fig. 3. Dimensionless load-carrying capacity $\overline{W}$ for various values of permeability parameter φ at $\bar{l} = 0.2$ and $C_0 = 0.01$.

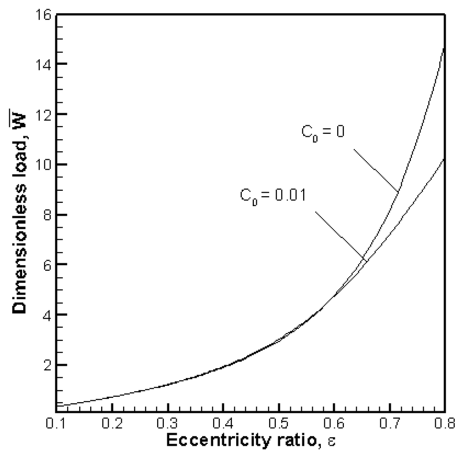

Fig. 4. Dimensionless load capacity $\overline{W}$ for various values of elastic coefficient C_0 at $\bar{l} = 0.2$ and $\varphi = 5\times10^{-6}$.