A numerical modeling of natural convection of a Cu-water nanofluid in a square enclosure containing different shapes of heating cylinders

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Abstract
This study examined numerically natural convection heat transfer of a Cu– water nanofluid in a square enclosure containing a central heating cylinder. A two-dimensional solution for natural convection was obtained using the finite volume method with SIMPLE algorithm. A detailed parametric study is conducted to investigate the effect of three different geometric forms of the inner cylinder (circular, square and triangular) on fluid flow and heat transfer characteristics for a wide range of Rayleigh number from 10^3 to 10^6 and volume fraction of nanoparticles from 0% to 5%. The results showed that, by changing the shape of the inner cylinder from circular to triangular, the heat transfer rate changes significantly. It is also found that by increasing the volume fraction of nanoparticles and Rayleigh number, the heat transfer rate increases.

Keywords : *Enclosure, Natural Convection, Heating cylinder, Nanofluid.*

1. Introduction
Natural convection fluid flow and heat transfer in an enclosure has practical relevance to several engineering applications such as heating and cooling nuclear systems of reactors, lubrication technologies, cooling of electronic devices, ventilation of rooms with radiators, cooling of containers and heat exchangers. One of the first works on the numerical simulation of natural convection inside cavities is the pioneering work of De Vahl Davis [1]. He performed a numerical simulation on a square cavity with two vertical isothermal walls, one cold and one hot, and two horizontal adiabatic walls. This cavity with those boundary conditions is known as differentially heated cavity (DHC).

El Abdallaoui et al. [2] performed a numerical simulation of natural convection between a decentered triangular heating cylinder and centered triangular heating cylinder in a square outer cylinder filled with a pure fluid or a nanofluid using the lattice Boltzmann method. The results indicate that the horizontal displacement from the centered position to decentered position leads to a considerable increase of heat transfer at weak Rayleigh and the vertical displacement has most important effect on heat transfer at high values of Rayleigh.

The aim of this study is to examine the heat transfer rate of natural convection heat transfer of nanofluid (Cu-water) in a two-dimensional square enclosure containing a central heating cylinder. The first case under investigation is characterized the numerical models used in our study. The computational procedure elaborated in this study is validated against the numerical results of other investigations. We studied the effects of different geometric forms of the heating cylinder (circular, square and triangular) which have the same peripheral area. Wide range of parameters such as Rayleigh number ($10^3 \leq Ra \leq 10^6$), and volume fraction of nanoparticles ($0 \leq \varphi \leq 0.05$) have been used. The new models of the thermal conductivity and effective viscosity investigated by Corcione et al. [3] are used to estimate themophysical proprieties of the nanofluid. Corcione et al [3] reported that the various parameters such as volume fraction, diameter and type of the nanoparticles and temperature of the nanofluid can significantly influence the effective viscosity and thermal conductivity of the nanofluid. Our numerical results are presented in the form of plots of isotherms, streamlines and average Nusselt numbers to show the influence of nanofluid and pertinent parameters.

2. Problem statement
The studied configurations and coordinate system of the considered enclosure in the present study are shown in Fig. 1. Two vertical walls are cooled at constant temperatures(T_c) the other walls are thermally insulated. The enclosure is a square with height H containing a central heating cylinder with different geometric forms (circular, square and triangular) which have the same peripheral area. The diameter of circular heating cylinder is equal to (w), so the peripheral area can be found which is equal to that of square and triangular cylinder. These cylinders are maintained at hot temperature (T_h). The fluid in the enclosure is a water-based nanofluid containing Cu nanoparticles. It is assumed that the nanofluid is newtonian, incompressible and laminar and the base fluid and the nanoparticles are in a thermal equilibrium state. The thermo-physical properties of the nanofluid used in this study are evaluated at the average fluid temperature $(T_c + T_h)/2$ as listed in Table 1.

Table 1 Thermo-physical properties of water and nanoparticles at T = 300 K .

	Copper (Cu)	Water (H_2O)
C_p(J/Kg K)	385	4179
ρ(Kg/m^3)	8933	997.1
k(W/mK)	401	0.6
β(K^{-1})	1.67 10^{-5}	27.6 10^{-5}
μ(kg m^{-1} s^{-1})	–	0.855 10^{-3}
$d_p(nm)$	25	0.385

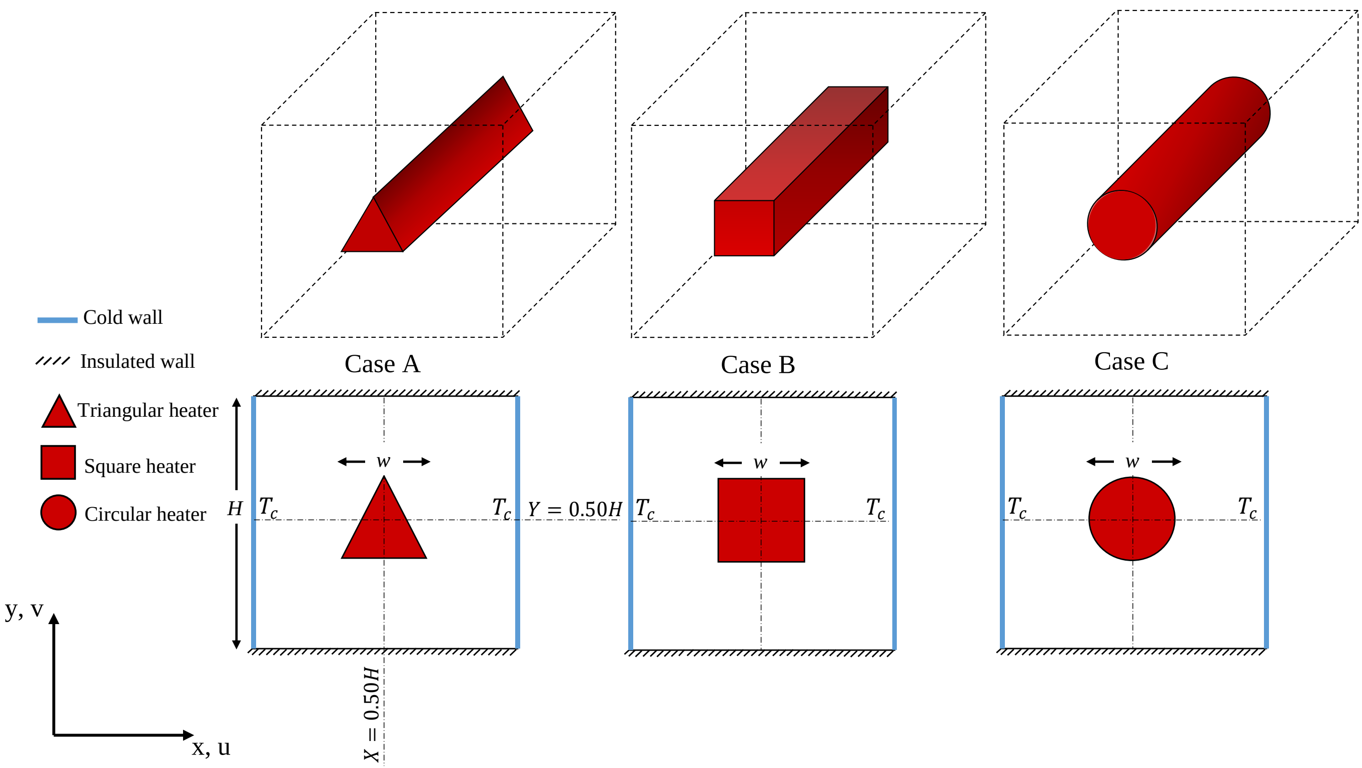

Fig. 1 Schematic of the enclosure containing a central heating cylinder with different geometric forms and boundary conditions.

3. Mathematical formulation

The governing equations (1)–(4) are written in the following dimensionless form:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (1)$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\frac{\rho_f}{\rho_{nf}}\frac{\mu_{nf}}{\mu_f}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \qquad (2)$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial p}{\partial Y}+Pr\frac{\rho_f}{\rho_{nf}}\frac{\mu_{nf}}{\mu_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra.Pr\frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}\,\theta \qquad (3)$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (4)$$

Dimensionless form of the boundary conditions can be written as:

$$\begin{array}{ll} U=0,\ V=0,\ \partial\theta/\partial Y=0 & \text{on bottom wall} \\ U=0,\ V=0,\ \partial\theta/\partial Y=0 & \text{on upper wall} \\ U=0,\ V=0,\ \theta=0 & \text{on right wall} \\ U=0,\ V=0,\ \theta=0 & \text{on left wall} \end{array} \qquad (5)$$

The total mean Nusselt number of all cavity's wall is defined as:

$$\overline{Nu}_{tot}=\frac{1}{H}\int_0^H\frac{k_{nf}(\varphi)}{k_f}\left\{\left|\frac{\partial\theta}{\partial X}\right|_{left}+\left|\frac{\partial\theta}{\partial X}\right|_{right}\right\}dY \qquad (6)$$

4. Results and discussion

In the present investigation, numerical results of natural convection heat transfer of a Cu-water nanofluid in a square enclosure containing a central heating cylinder with different geometric forms (circular, square and triangular) are numerically simulated. The numerical results are presented in terms of streamlines, isotherms and mean Nusselt number for various values of Rayleigh number ($10^3 \le Ra \le 10^6$) and volume fraction of the nanoparticles ($0 \le \varphi \le 0.05$), while the size of each different inner cylinders (w) is fixed at 0.4 and the Prandtl number of the pure water ($Pr = 6.2$).

Fig. 2 illustrates the effect of different geometric forms of the inner cylinder on the streamlines and isotherms for various values of Rayleigh number ($10^3 \le Ra \le 10^6$) and volume fraction of the nanoparticles (φ =0%, 5%). The enclosure is filled by a Cu- water nanofluid (φ = 5%), for comparisons, the streamlines and the isotherms for pure fluid and nanofluid are shown by dashed line and solid line respectively. Streamlines in Fig. 2(a) demonstrates that in all cases (A, B and C), two overall rotating symmetric eddies are formed by the fluid temperature differences, one clockwise vortex on the right and one counter-clockwise vortex on the left. The isotherms in Fig. 2(b) show the characteristics of conduction dominated regime since they are uniformly distributed and also take the shape of the heating cylinders. It can be seen that the behavior of isotherms is symmetrical to the vertical center line. As the Rayleigh number increases to 10^5, a plume starts to appear on the top of the heating cylinder, the presence of high-temperature gradients across this plum, makes the upper part much more intense concerning the thermal gradient than the lower part. It is clear that the isotherms start to deform from this pattern and distinct thermal boundary layers are formed around the heating cylinder which means that the heat transfer mechanism is changing from conduction to the convection regime.

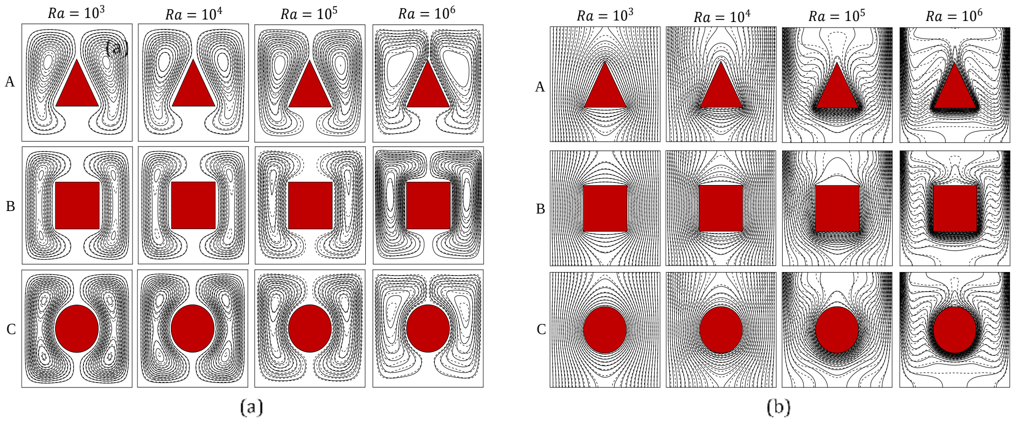

Fig. 2 The (a) streamlines and (b) isotherms inside the enclosure corresponding to different geometric forms of the inner cylinder , with case of pure fluid (dashed line) and Cu-water (solid line) nanofluid with $\varphi = 0.05$ for different Rayleigh number.

The streamlines in Fig. 2(b) show that, by increasing the Rayleigh number and hence buoyancy force, the flow intensity increases and subsequently the core of the vortices move upward which means that the heat transfer mechanism is changing from conduction to the convection regime. As it is seen from Fig. 2(a-b), some differences in streamlines and isotherms of pure fluid and nanofluid. This can be attributed to the higher viscosity of nanofluid compared to that of the pure. However, at different Rayleigh number, by changing shape of the inner cylinder from case A (triangular) to C (circular), the vortices core change significantly which indicate that the geometric form of the cylinder effect on the heat transfer.

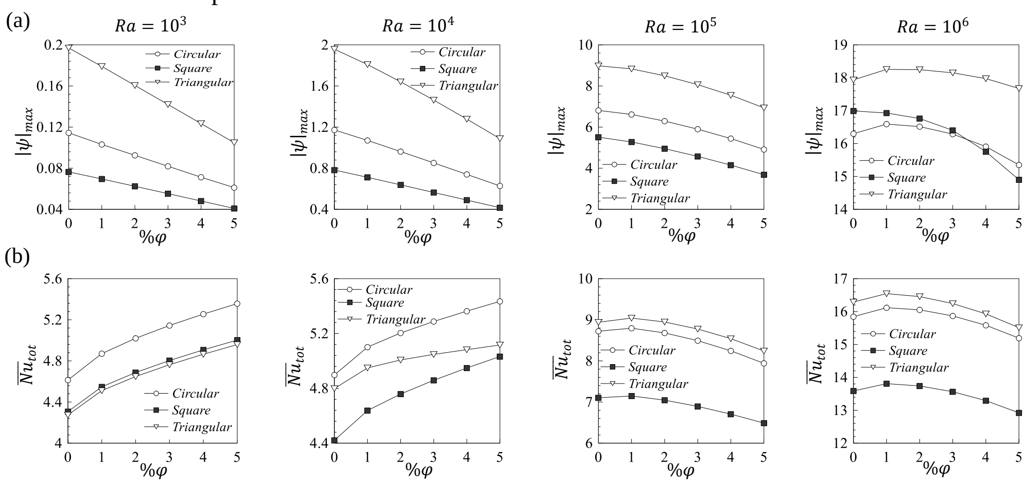

Fig. 3 Variation of (a) $|\psi|_{max}$ and (b) $\overline{Nu}_{tot}$ corresponding to different geometric forms of the inner cylinder for different Rayleigh numbers and volume fraction of the nanoparticles

Fig. 3(a-b) shows the effects of different geometric forms on $|\psi|_{max}$ and $\overline{Nu}_{tot}$ for different volume fraction of nanoparticles and Rayleigh numbers. Fig. 3(b) shows that increasing the Rayleigh number, and the volume fraction of nanoparticles, the rate of heat transfer increases. However, we can see from Fig. 3(b) that by changing the shape of the inner cylinder from triangular to circular, the heat transfer rate changes significantly. At small Rayleigh number (Ra = 10^3 and 10^4), the maximum values of the mean Nusselt number are deduced for the circular cylinder. By increasing Rayleigh number beyond 10^4, we can clearly observe that the triangular cylinder is the best for improving the heat transfer rate, due to low resistance to fluid circulation within the enclosure compared to other geometric shapes (circular and square).

5. Conclusion

According to the presented results, the following conclusions are drawn:

i. By increasing the Rayleigh number, and the volume fraction of nanoparticles, the rate of heat transfer increases.
ii. At small Rayleigh number, the maximum values of the mean Nusselt number are deduced for the circular cylinder.
iii. By increasing Rayleigh number beyond 10^4, the triangular cylinder is the best for improving the heat transfer rate.

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